

CONTENTS

PART I. INTRODUCTORY RESULTS

1. Arithmetical functions	1
2. Some sum functions	6
3. Characters	10
4. Pólya's theorem	14
5. Dirichlet series	18
6. Schinzel's hypothesis	23
7. The large sieve	28
8. The upper-bound sieve	32
9. Franel's theorem	36

PART II. THE PRIME-NUMBER THEOREM

10. A modular relation	40
11. The functional equations	45
12. Hadamard's product formula	50
13. Zeros of $\xi(s)$	55
14. Zeros of $\xi(s, \chi)$	58
15. The exceptional zero	61
16. The prime-number theorem	66
17. The prime-number theorem for an arithmetic progression	70

PART III. THE NECESSARY TOOLS

18. A survey of sieves	73
19. The hybrid sieve	79
20. An approximate functional equation (I)	84
21. An approximate functional equation (II)	89
22. Fourth powers of L -functions	93

CONTENTS

PART IV. ZEROS AND PRIME NUMBERS

23. Ingham's theorem	98
24. Bombieri's theorem	103
25. I. M. Vinogradov's estimate	107
26. I. M. Vinogradov's three-primes theorem	110
27. Halász's method	114
28. Gaps between prime numbers	118
NOTATION	123
BIBLIOGRAPHY	124
INDEX	127