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Chapter 1. **Logic, Set Theory, and the Axiom of Choice** 1

This review chapter surveys elementary logic and set theory, including such concepts as countability, power set, etc. The axiom of choice is presented, along with its most useful variants. The chapter closes with an outline of category theory.

Chapter 2. **Metric Spaces** 15

After a review of the real numbers, Euclidean spaces are studied in detail. This is followed by a treatment of all the basic properties of metric spaces and continuous functions from a metric space to another.

Chapter 3. **General Topological Spaces. Bases. Continuous Functions. Product Spaces** 35

General topological spaces are treated here. Concepts such as interior, exterior, boundary, closure, etc., are examined in detail. Bases and subbases are set up, and the relative and product topologies are defined. After a treatment of continuous functions, the chapter closes with the notion of a homeomorphism.

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The Hausdorff separation axiom is introduced here; compact spaces are examined from various viewpoints, and the compact subspaces of a Euclidean space are found. More, generally, compact metric spaces are characterized by the Bolzano-Weierstrass property. The Tychonoff theorem is then proved. The chapter ends with a study of the basic properties of connected and arcwise connected spaces.

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The first and second axioms of countability are introduced, as well as the notion of separability. The role of sequences in the topology of first countable spaces is established. Normal spaces are treated, followed by Urysohn's lemma, and finally the Urysohn metrization theorem.

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This chapter surveys a variety of topics which are not in the main line of the earlier presentation, but most of which are applied later in the book. Included at this point are such topics as one-point compactifications and the uniform convergence of functions.

Chapter 7. **Generalities about Manifolds; The Classification of Surfaces** 105

Manifolds are defined, and the basic properties, including an elementary imbedding theorem, are proved. Projective spaces and spheres are the beginning examples. Topological groups, Lie groups, and differentiable manifolds are briefly surveyed. The remainder of the chapter is devoted to a classification of compact surfaces.

Chapter 8. **The Fundamental Group** 138

This is the first time where an algebraic invariant is introduced to study topological problems. The construction and basic properties occupy most of the chapter. At the end, the fundamental group is studied for Cartesian products and topological groups.

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Covering spaces are defined, and the questions of lifting paths and homotopies are treated. This yields the classification of covering spaces in terms of subgroups of the fundamental group. The existence of covering spaces for semi-locally simply-connected spaces is next. The chapter ends with some computational examples.

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The fundamental group is studied for a connected simplicial complex. After some preliminaries, the fundamental group is proven to be isomorphic to the edge-path group. Various applications and results on the general structure of the fundamental group follow.

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