

Contents

NOTATION	xxi
CHAPTER	
1 THE TOPOLOGICAL SPACES \mathcal{F} , \mathcal{G} , \mathcal{K} , AND $\overline{\mathcal{K}}$.	1
1-1 General Hypotheses and Notations.	1
1-2 The Compact Space $\mathcal{F}(E)$.	3
Convergence in \mathcal{F} .	6
The Operations $\underline{\lim}$ and $\overline{\lim}$.	7
Upper and Lower Semicontinuities.	8
The Case of a Metric Space E .	10
1-3 The Spaces \mathcal{G} and \mathcal{K} .	11
1-4 The Space $\mathcal{K}(E)$ and the Myope Topology.	12
The Hausdorff Metric.	15
1-5 Case of the Euclidean Space $E = \mathbf{R}^d$.	16
Minkowski's Addition and Subtraction.	16
Opening and Closing A by B .	18
Convexity.	21
Compact Sets Infinitely Divisible for \oplus .	22
Granulometries.	24
2 THE RANDOM CLOSED SETS (RACS).	27
2-1 The Functional T Associated with a RACS.	27
2-2 The Choquet Theorem.	30
2-3 Conditional RACS.	35
Independent RACS.	40
Inductive Limit of RACS.	40

2-4	The Point of View of the Space Law.	40
2-5	A.S. Continuity, P -Continuity, and Measurability.	45
2-6	Open Random Sets, and Random Sets on \mathcal{K} .	48
2-7	Examples.	49
	Granulometries.	51
3	INDEFINITELY DIVISIBLE RANDOM CLOSED SETS (IDRACS).	54
3-1	Characterization of the Functional Q Associated with an IDRACS.	54
3-2	Poisson Processes and σ -Finite Measures on \mathcal{F}' .	57
3-3	Stationary Boolean Models in a Euclidean Space.	61
3-4	Stable RACS in a Euclidean Space.	62
3-5	Poisson Flats in a Euclidean Space.	65
	Results Concerning the Space $C(\mathcal{K}')$.	67
	Random Cross Sections of a Compact Set B .	70
	Poisson Network Induced on a Linear Variety.	71
	Point Processes Induced on the $(d-k)$ -dimensional Varieties.	72
	Networks Induced on the Varieties with Dimension $< d-k$.	73
4	CONVEXITY.	75
4-1	The Minkowski Functionals.	79
	The Steiner Formula.	81
	The Crofton Formula.	82
4-2	The RACS Almost Surely Convex.	82
4-3	The Linear Granulometries.	85
	The Random Intercepts.	87
4-4	The Convex Cone $\mathcal{R} = C_0(\mathcal{K})$.	88
	The Ordering \geq in $\mathcal{R} = C_0(\mathcal{K})$.	91
	The Minkowski Space $M(S_0)$.	92
	Extension of a Positively Linear Functional on \mathcal{R} .	92
4-5	The Convex Cone \mathcal{R}_1 and the Steiner Class.	93
	The Uniqueness Theorem.	94
	Geometrical Interpretation.	98
	Characterization of a Line or Hyperplane Poisson Network.	98
	The Surface Measure G_{d-1}^K .	99
	An Extension of the Steiner Formula.	101

4-6	The Random Variables $ S, S_p $. 105	
	The Random Measure Associated with a Poisson Flat Network. 107	
	The Covariance Measure. 108	
4-7	The Minkowski Measures. 111	
	The Random Version of the Minkowski Measures. 116	
	The Convex Ring \mathfrak{C} . 117	
	Extension of the Minkowski Measures. 119	
	The Convexity Number. 122	
	The Generalized Random Minkowski Measures. 124	
	Further Generalization. 127	
5	SEMI-MARKOVIAN RACS IN \mathbb{R}^d .	130
5-1	The Semi-Markovian Property. 130	
5-2	Stationary Semi-Markovian RACS on \mathbb{R} . 133	
5-3	The Boolean Models with Convex Grains. 137	
	Boolean Models Induced on the Lines. 140	
	The Densities of the Minkowski Functionals. 142	
	Boolean Models Induced on Linear Varieties. 144	
5-4	The Stationary SMIDRACS. 146	
	SMIDRACS Induced on the Lines. 150	
	The Densities of the Minkowski Functionals. 152	
	SMIDRACS Induced on Linear Varieties. 153	
	The Isotropic Case. 153	
6	POISSON HYPERPLANES AND POLYHEDRA.	155
6-1	Stationary Poisson Hyperplane Networks. 156	
	The Densities of the $(d-k)$ -Volumes. 157	
	Calculation of $\nu_k, k=1, 2, \dots, d$. 158	
	The Covariance Measures (Isotropic Case). 161	
6-2	The Poisson Polyhedra and the Conditional Invariance. 164	
	The Conditional Invariance. 166	
	The Number Law. 168	
6-3	Applications. 173	
	Expectations of the Minkowski Functionals $W_k(\Pi)$. 174	
	The Granulometry with Respect to the Unit Ball. 175	
	A Relationship Between the Law of the Volume V and the Law of the Surface Area S . 176	

	A Relationship Between $\psi(\Pi_0)$ and the Number of $(d-1)$ Faces. 177	
	The First Moments of the Volume V . 178	
	The First Moments of the Projection Area V' . 180	
	The First Moments of the Surface Area S . 181	
	The Isotropic Poisson Polygons. 182	
7	THE GRANULOMETRIES.	186
7-1	Algebraic Openings and Closings. 186	
	Extensions of an Increasing Mapping. 187	
	τ -Openings and τ -Closings in \mathbb{R}^d . 189	
7-2	The Granulometries. 192	
	Regularization of a Granulometry. 193	
	Critical Elements of a Granulometry. 194	
	Euclidean Granulometries. 195	
	Euclidean Granulometries U.S.C. and Compact. 198	
	Other Examples. 200	
7-3	Granulometry of a RACS and of Its Complementary Set. 200	
	The Size Distribution of the Pores. 201	
7-4	Openings and Granulometries (General Case). 203	
	Openings and Closings U.S.C. on \mathcal{F} or \mathcal{K} . 203	
	Smallest U.S.C. Upper Bound of an Opening or a Closing on \mathcal{K} . 205	
	Compact Openings. 208	
	Granulometries U.S.C. and Compact. 210	
7-5	Openings and Closings L.S.C. on \mathcal{K} or \mathcal{F} . 212	
8	INCREASING MAPPINGS.	217
8-1	Algebraic Properties of (Increasing) τ -Mappings. 217	
8-2	Topological Properties of τ -Mappings. 222	
	Extension of a Mapping U.S.C. on \mathcal{K} . 224	
	τ -Mappings Compatible with \cup or \cap . 226	
8-3	Topological Complements. 228	
8-4	Point of View of the Inverse Mappings. 231	
9	INTEGRALS AND MEASURES VALUED IN \mathcal{K}_0 .	235
9-1	The Riemann-Minkowski Integral. 235	
	The Convexity of the Riemann-Minkowski Integral. 238	

The Stieltjes-Minkowski Integral. 239

9-2 Radon Measures Valued in $\mathcal{K}_0(\mathbf{R}^d)$. 240

 The Spaces Φ_k , Φ_g , and $\mathcal{C}_{\mathcal{K}}^+$. 240

 Pseudo-Integrals Valued in $\mathcal{K}_0(\mathbf{R}^d)$. 242

 Extension on Φ_g of a Pseudo-Integral. 243

 Extension on Φ_k . 244

 The Space Φ of the Pseudo-Integrable Functions. 245

 The \mathcal{K}_0 -Integrals. 250

9-3 Abstract Measures Valued in $\overline{\mathcal{K}_0}$. 251

 The Integral Associated with a \mathcal{K}_0 -Measure. 252

BIBLIOGRAPHY 254

INDEX 257