

# CONTENTS

Preface *ix*

Acknowledgments *xi*

## Chapter 1 INTRODUCTION

1.1 Euclidean Planes *1*

1.2 Incidence Bases *6*

1.3 Set Theory *8*

## Chapter 2 AFFINE PLANES

2.1 Axioms for an Affine Plane *12*

2.2 Examples *17*

## Chapter 3 PROJECTIVE PLANES

3.1 Axioms for a Projective Plane *28*

3.2 Examples *33*

3.3 Algebraic Incidence Bases *37*

3.4 Self-Dual Axioms *40*

## Chapter 4 AFFINE AND PROJECTIVE PLANES

4.1 Isomorphism *45*

4.2 Deletion Subgeometries *49*

4.3 The Imbedding Theorem *54*

**Chapter 5 THEOREMS OF DESARGUES AND PAPPUS**

- 5.1 Configurations 61
- 5.2 Theorem of Desargues 64
- 5.3 Theorem of Pappus 73

**Chapter 6 COORDINATIZATION**

- 6.1 Coordinates 80
- 6.2 Addition 82
- 6.3 Multiplication 89
- 6.4 Algebraic Systems and Incidence Bases 98
- 6.5 The Coordinatization Theorem 100
- 6.6 Finite Projective Planes 104

**Chapter 7 PROJECTIVITIES**

- 7.1 Perspectivities and Projectivities 106
- 7.2 Some Classical Theorems 114
- 7.3 A Nonpappian Example 118

**Chapter 8 HARMONIC QUADRUPLES**

- 8.1 Fano Axiom 122
- 8.2 Harmonic Quadruples 124

**Chapter 9 THE REAL PROJECTIVE PLANE**

- 9.1 Separation 133
- 9.2 Ordered Fields 138
- 9.3 Completeness and the Real Numbers 142
- 9.4 Separation for Basis 3.5 147
- 9.5 The Real Projective Plane 154
- 9.6 Euclidean Planes 157

**Chapter 10 PROJECTIVE SPACES—Part 1**

- 10.1 Axioms for a Projective Space 162
- 10.2 Examples 168

**Chapter 11 PROJECTIVE SPACES—Part 2**

- 11.1 Subspaces and Dimension 172
- 11.2 Intervals and Complements 186
- 11.3 Dual Spaces 191

**Appendix A HILBERT'S AXIOMS FOR A EUCLIDEAN PLANE**

- Group I. Axioms of Connection 195
- Group II. Axioms of Order 196
- Group III. Axiom of Parallels 197
- Group IV. Axioms of Congruence 197
- Group V. Axiom of Continuity 198

**Appendix B DIVISION RINGS 199****Appendix C QUATERNIONS 205****REFERENCES 211****Index of Special Symbols 213****Subject Index 217**