## Contents

I. Elementary Set Theory

| 1. | Sets and Subsets                         |
|----|--|
| 2. | Unions and Intersections 5               |
| 3. | Binary Relations: Cartesian Products and |
|    | Mappings 8                               |
| 4. | Infinite Sets 13                         |
| 5. | Set Theory and the Foundations of        |
|    | Mathematics 17                           |
|    | References 19                            |

## 2. Topological Spaces

21

- Basic Concepts 21
- 2. Convergence of a Sequence 30
- 3. Bases and Subbases for a Topology 33
- 4. Subspaces and Separated Sets 38
- 5. Connected sets 40
- 6. Connected Subsets of R 43
- 7. Metric Spaces 46
- 8. Other Approaches to a Topological Space 52

| ×  | CONTENTS  |     |  |
|----|---|-----|--|
| 3. | Mappings of Topological Spaces  |     |  |
|    | <ol> <li>Continuous Functions 58</li> <li>Restrictions and Extensions of Mappings 64</li> <li>Invariants Under Continuous Mappings 66</li> <li>Homeomorphisms 69</li> <li>Some Comments on Topology 71</li> </ol>   |     |  |
| 4. | Compactness   | 76  |  |
|    | <ol> <li>Conditions Related to Compactness 76</li> <li>Compact Metric Spaces 81</li> <li>Uniform Continuity 84</li> <li>Compact Subsets of R 85</li> <li>Separation Properties 87</li> <li>Compactness and Separation Properties 90</li> <li>One-point Compactification 92</li> </ol> |     |  |
| 5. | Product Spaces  | 95  |  |
|    | <ol> <li>The Product of Two Spaces 95</li> <li>The Product of n Spaces 97</li> <li>General Product Spaces 101</li> </ol>  |     |  |
| 6. | Metric Spaces   | 110 |  |
|    | <ol> <li>Complete Normality of Metric Spaces 110</li> <li>Products of Metric Spaces 112</li> <li>Complete Metric Spaces 115</li> <li>A Metrization Theorem 121</li> </ol>   |     |  |
| 7. | More on Product Spaces and Function Spaces  | 125 |  |
|    | <ol> <li>Compactness in Terms of Subbases 125</li> <li>The Tychonoff Theorem 128</li> <li>Tychonoff Cubes 129</li> <li>Tychonoff Spaces 132</li> </ol>  |     |  |

|    | 5.  | A Second Proof of Urysohn's Metrization          |     |
|----|-----|--|-----|
|    |     | Theorem 134                                      |     |
|    | 6.  | Ordinal Numbers 135                              |     |
|    | 7.  | Spaces Involving Ordinal Numbers 141             |     |
|    | 8.  | Function Spaces 144                              |     |
| 8. | N   | ets and Convergence                              | 148 |
|    | ı.  | Directed Sets and Nets 149                       |     |
|    | 2.  | Convergence of a Net in a Space 149              |     |
|    | 3.  | Compactness in Terms of Nets 152                 |     |
|    | 4.  | Topologies Determined by Nets 153 References 155 |     |
| 9. | Pe  | eano Spaces                                      | 156 |
|    | ı.  | Continuous Curves 156                            |     |
|    | 2.  | Continua and Cut Points 158                      |     |
|    | 3.  | The Arc and the Simple Closed Curve 161          |     |
|    | 4.  | Arcwise Connectivity 163                         |     |
|    | 5.  | The Cantor Ternary Set 166                       |     |
|    | 6.  | The Hahn-Mazurkiewicz Theorem 168                |     |
|    | Inc | dex  | 171 |
|    |     |  |     |