

# Contents

<b>1</b>	<b>THE NATURAL NUMBERS</b>	<b>1</b>
1-1	Introduction	1
1-2	Some Properties of Numbers	2
1-3	Some Deeper Questions	5
1-4	The Nature of Mathematics	5
1-5	Peano's Postulates	9
1-6	Some Preliminary Theorems	13
1-7	Addition	14
1-8	Multiplication	20
<b>2</b>	<b>RELATIONS</b>	<b>25</b>
2-1	Introduction	25
2-2	Binary Operation	25
2-3	Cartesian Products	27
2-4	Matrices and Relations	29
2-5	Equivalence Relations	31
2-6	Ordering	34

<b>3</b>	<b>INTEGERS</b>	<b>37</b>
3-1	Introduction	37
3-2	Ordered Pairs of Natural Numbers	37
3-3	An Equivalence Relation	40
3-4	Definition of the Integers	43
3-5	Theorems concerning Integers	45
3-6	Linear Ordering	50
<b>4</b>	<b>ABSTRACT ALGEBRA</b>	<b>53</b>
4-1	Introduction	53
4-2	Groups and Semigroups	54
4-3	Rings and Integral Domains	60
4-4	Fields	64
4-5	Isomorphism and Homomorphism	67
<b>5</b>	<b>RATIONAL NUMBERS</b>	<b>73</b>
5-1	Introduction	73
5-2	Ordered Pairs of Integers	73
5-3	Theorems concerning Rational Numbers	76
5-4	Two Important Isomorphisms	79
5-5	Ordering	81
5-6	Algebraic Structure of the Rational Numbers	84
<b>6</b>	<b>REAL NUMBERS</b>	<b>85</b>
6-1	Introduction	85
6-2	Cuts	86
6-3	Addition of Cuts	89
6-4	Multiplication of Positive Cuts	92
6-5	Multiplication of Cuts	95
6-6	Algebraic Structure of Cuts	98
6-7	Further Properties of Cuts	99

<b>7</b>	<b>EXTENSIONS OF THE REAL NUMBER FIELD</b>	<b>103</b>
	7-1 Introduction	103
	7-2 Complex Numbers	103
	7-3 Quaternions	107
	7-4 Vector Spaces	110
	<b>ANSWERS TO ODD-NUMBERED EXERCISES</b>	<b>113</b>
	<b>INDEX</b>	<b>135</b>