

TABLE OF CONTENTS

PART I COMBINATORIAL GEOMETRY	1
CHAPTER 1 Fundamental Concepts in Combinatorial Geometry	3
1.1. Arrangements of Hyperplanes	4
1.2. Counting Faces and Incidences	6
1.3. Combinatorial Equivalence	10
1.4. Configurations of Points	12
1.5. Sylvester's Problem	15
1.6. Convex Polytopes and Convex Polyhedra	16
1.7. Zonotopes	20
1.8. Voronoi Diagrams	23
1.9. Exercises and Research Problems	25
1.10. Bibliographic Notes	27
CHAPTER 2 Permutation Tables	29
2.1. Circular Sequences	29
2.2. Encoding Arrangements and Configurations	32
2.3. A Circularly Non-Realizable 5-Sequence	35
2.4. Arrangements of Pseudo-Lines	37
2.5. Some Combinatorial Problems in the Plane	39
2.6. Exercises and Research Problems	41
2.7. Bibliographic Notes	44
CHAPTER 3 Semispaces of Configurations	45
3.1. Semispaces and Arrangements	46
3.2. k -Sets and Levels in Arrangements	47
3.3. A Lower Bound on the Number of Bisections in the Plane	50
3.4. Lower Bounds on the Number of k -Sets in the Plane	52
3.5. Extensions to Three and Higher Dimensions	54
3.6. Semispaces and Circular Sequences	55
3.7. An Upper Bound on the Number of k -Sets in the Plane	58
3.8. Exercises and Research Problems	60
3.9. Bibliographic Notes	61

CHAPTER 4 Dissections of Point Sets	63
4.1. Centerpoints	63
4.2. Ham–Sandwich Cuts	66
4.3. Erasing Subdivisions in the Plane	70
4.4. Simultaneous Four–Section in Three Dimensions	73
4.5. Erasing Cell Complexes in Three Dimensions	77
4.6. Generalizations to Higher Dimensions	78
4.7. Exercises and Research Problems	79
4.8. Bibliographic Notes	81
CHAPTER 5 Zones in Arrangements	83
5.1. Supported Cells, Zones, and Duality	84
5.2. Sweeping a Simple Arrangement	86
5.3. Tight Bounds in the Plane	89
5.4. Asymptotically Tight Bounds in d Dimensions	93
5.5. An Implication of the Result on Zones	94
5.6. Exercises and Research Problems	95
5.7. Bibliographic Notes	96
CHAPTER 6 The Complexity of Families of Cells	97
6.1. Definitions and Preliminary Results	98
6.2. The Complexity of a Polytope	99
6.2.1. Cyclic Polytopes	100
6.2.2. Euler’s Relation	102
6.2.3. The Dehn–Sommerville Relations	104
6.2.4. An Asymptotic Version of the Upper Bound Theorem	106
6.3. The Complexity of a Few Cells in Two Dimensions	107
6.4. Lower Bounds for Moderately Many Cells	109
6.5. Lower Bounds for Many Cells	111
6.6. Upper Bounds for Many Cells	114
6.7. Exercises and Research Problems	116
6.8. Bibliographic Notes	118
PART II FUNDAMENTAL GEOMETRIC ALGORITHMS	121
CHAPTER 7 Constructing Arrangements	123
7.1. Representing an Arrangement in Storage	123
7.2. The Incremental Approach	125
7.3. Initiating the Construction	126
7.4. Geometric Preliminaries	128
7.5. Incrementing the Arrangement	130
7.6. The Analysis of the Algorithm	134
7.7. Exercises and Research Problems	136

7.8.	Bibliographic Notes	137
CHAPTER 8	Constructing Convex Hulls	139
8.1.	Convex Hulls and Duality	140
8.2.	The Incidence Graph of a Convex Polytope	141
8.3.	Two Algorithms in Two Dimensions	142
8.3.1.	The Beneath–Beyond Method	143
8.3.2.	Using Divide-and-Conquer	145
8.4.	The Beneath–Beyond Method in d Dimensions	147
8.4.1.	Geometric Preliminaries	148
8.4.2.	Towards the Incrementation of the Convex Hull	152
8.4.3.	Pyramidal Updates	153
8.4.4.	Non-Pyramidal Updates	154
8.4.5.	The Analysis of the Algorithm	156
8.5.	Divide-and-Conquer in Three Dimensions	158
8.5.1.	Coping with Degenerate Cases	159
8.5.2.	The Upgraded Incidence Graph	160
8.5.3.	Geometric Preliminaries	162
8.5.4.	Wrapping Two Convex Polytopes	167
8.5.5.	The Analysis of the Algorithm	172
8.6.	Exercises and Research Problems	173
8.7.	Bibliographic Notes	175
CHAPTER 9	Skeletons in Arrangements	177
9.1.	Skeletons and Eulerian Tours	178
9.2.	Towards the Construction of a Skeleton	181
9.3.	Constructing a Skeleton in a Simple Arrangement	183
9.4.	Simulating Simplicity	185
9.4.1.	A Conceptual Perturbation	186
9.4.2.	Simulating the Perturbation	188
9.4.3.	Computing the Sign of a Determinant of Polynomials	190
9.5.	Penetration Search and Extremal Queries	192
9.5.1.	Extremal Queries in the Plane	193
9.5.2.	Extremal Queries in Three Dimensions: the Data Structure	195
9.5.3.	Extremal Queries in Three Dimensions: the Query Algorithm	201
9.5.4.	Dynamic Extremal Search	202
9.6.	Exercises and Research Problems	205
9.7.	Bibliographic Notes	208
CHAPTER 10	Linear Programming	209
10.1.	The Solution to a Linear Program	210
10.2.	Linear Programming and Duality	212
10.3.	Linear Programming in Two Dimensions	214

10.3.1. Prune: Eliminate Redundant Half-Planes	215
10.3.2. Bisect: Decrease the Range of the Linear Program	217
10.3.3. Find_Test: Determine the Median	219
10.3.4. Assembling the Algorithm	220
10.4. Linear Programming in Three and Higher Dimensions	223
10.4.1. The Geometry of Pruning	224
10.4.2. The Geometry of Bisecting	225
10.4.3. Searching Lines in the Plane	228
10.4.4. The Geometry of Searching	230
10.4.5. The Overall Algorithm	234
10.5. Exercises and Research Problems	236
10.6. Bibliographic Notes	238
CHAPTER 11 Planar Point Location Search	241
11.1. Euler's Relation for Planar Graphs	242
11.2. The Geometry of Monotone Subdivisions	244
11.3. A Tree of Separators for Point Location	248
11.4. Representing a Plane Subdivision	251
11.5. Constructing a Family of Separators	252
11.6. Optimal Search by Connecting Separators	256
11.7. Constructing the Layered DAG	258
11.8. Refining Non-Monotone Subdivisions	260
11.9. Exercises and Research Problems	262
11.10. Bibliographic Notes	265
PART III GEOMETRIC AND ALGORITHMIC APPLICATIONS	269
CHAPTER 12 Problems for Configurations and Arrangements	271
12.1. Largest Convex Subsets	272
12.2. The Visibility Graph for Line Segments	275
12.3. Degeneracies in Configurations	278
12.4. Minimum Measure Simplices	282
12.5. Computing Ranks: Sorting in d Dimensions?	284
12.6. A Vector-Sum Maximization Problem	286
12.7. Exercises and Research Problems	288
12.8. Bibliographic Notes	290
CHAPTER 13 Voronoi Diagrams	293
13.1. Classical Voronoi Diagrams	294
13.2. Applications in the Plane	298
13.2.1. The Post Office Problem	298
13.2.2. Triangulating Point Sets	299
13.2.3. Delaunay Triangulations from Convex Hulls	303

13.2.4. Finding Closest Neighbors	306
13.2.5. Minimum Spanning Trees	306
13.2.6. Shapes of Point Sets	309
13.3. Higher-Order Voronoi Diagrams	315
13.4. The Complexity of Higher-Order Voronoi Diagrams	319
13.5. Constructing Higher-Order Voronoi Diagrams	324
13.6. Power Diagrams	327
13.7. Exercises and Research Problems	328
13.8. Bibliographic Notes	332
CHAPTER 14 Separation and Intersection in the Plane	335
14.1. Constructing Ham-Sandwich Cuts in Two Dimensions	336
14.1.1. Ham-Sandwich Cuts and Duality	336
14.1.2. Testing a Line	339
14.1.3. Finding Test Lines and Pruning	341
14.1.4. The Overall Algorithm	343
14.2. Answering Line Queries	345
14.2.1. The Ham-Sandwich Tree	345
14.2.2. Point Location in Arrangements	348
14.3. A Self-Dual Intersection Problem	349
14.4. Triangular Range Queries	351
14.5. Exercises and Research Problems	354
14.6. Bibliographic Notes	356
CHAPTER 15 Paradigmatic Design of Algorithms	359
15.1. The Problem: Stabbing Line Segments in the Plane	359
15.2. Geometric Transformation	361
15.3. Combinatorial Analysis	363
15.4. Divide-and-Conquer	365
15.5. Incremental Construction	367
15.6. Prune-and-Search	370
15.7. The Locus Approach	371
15.8. Dynamization by Decomposition	373
15.9. Exercises and Research Problems	376
15.10. Bibliographic Notes	378
REFERENCES	381
APPENDIX A Definitions	395
APPENDIX B Notational Conventions	409
INDEX	417