Contents

Preface		
Notation	xv	
0 Preliminaries	1	
0.1 Introduction	1	
0.2 Sums of multiplicative functions	1	
0.3 Sieve estimates	4	
0.4 Local distributions of prime factors and Poisson variables	4	
0.5 A general principle	6	
Notes on Chapter 0	9	
Exercises on Chapter 0	11	
1 The normal distribution of the prime factors	14	
1.1 The law of the iterated logarithm	14	
1.2 The normal size of $p_i(n)$ and $d_i(n)$	21	
Notes on Chapter 1	24	
Exercises on Chapter 1	25	
2 Sieving by an interval	27	
2.1 Introduction	27	
2.2 Statement of results concerning $H(x, y, z)$	29	
2.3 More applications	31	
2.4 Proof of Theorem 22	34	
2.5 Proof of Theorem 21(i) – small z	38	
2.6 Proof of Theorem 21(ii), (iii) - upper bounds	40	
2.7 Proof of Theorem 21(ii), (iii) - lower bounds	41	
2.8 Proof of Theorem 21(iv) – large z	46	
Notes on Chapter 2	48	
Exercises on Chapter 2	49	
3 Imaginary powers	52	
3.1 Introduction	52	

viii Contents

	3.2 A p.p. upper bound for $ \tau(n,\theta) $	52
	3.3 Ratios of divisors	56
	3.4 Average orders	57
	Notes on Chapter 3	62
	Exercises on Chapter 3	64
4	Measures of propinquity	65
	4.1 Introduction	65
	4.2 T and U – preliminary matters	65
	4.3 T and U – average orders	67
	4.4 The normal orders of $\log T$ and $\log U$	74
	4.5 The function $T(n,0)/\tau(n)$	79
	4.6 Erdős' function $\tau^+(n)$	83
	4.7 Hooley's function $\Delta(n)$	89
	Notes on Chapter 4	92
	Exercises on Chapter 4	93
5	Erdős' conjecture	95
	5.1 Introduction and results	95
	5.2 Proof of Theorem 50	99
	5.3 Proof of Theorem 51	104
	5.4 A p.p. upper bound for the Δ -function	111
	Notes on Chapter 5	118
	Exercises on Chapter 5	120
5	Hooley's Δ_r -functions – sharp bounds	121
	6.1 Introduction	121
	6.2 Lower bounds	122
	6.3 The critical interval	123
	6.4 Technical preparation	124
	6.5 Iteration inequalities	126
	6.6 Small y – the lower bound for Λ_r^-	127
	6.7 Fourier transforms – initial treatment	128
	6.8 Fourier transforms – an upper bound for Λ_r^+	132
	Notes on Chapter 6	136
	Exercises on Chapter 6	138
7	Hooley's Δ_r -functions – the critical interval	139
	7.1 Introduction	139
	7.2 Notation. The fundamental lemma	140
	7.3 First variant – differential inequalities	146

	Contents	ix
7.4 Second varian	t – double induction	150
Notes on Cha	pter 7	156
Exercises on C	Chapter 7	157
Appendix: Dis	tribution functions	158
Bibliography		161
Index		165