

Table of Contents

Chapter XIV. Evolution Problems: Cauchy Problems in \mathbb{R}^n

Introduction	1
§1. The Ordinary Cauchy Problems in Finite Dimensional Spaces	3
1. Linear Systems with Constant Coefficients	4
2. Linear Systems with Non Constant Coefficients	6
§2. Diffusion Equations	8
1. Setting of Problem	9
2. The Method of the Fourier Transform	10
3. The Elementary Solution of the Heat Equation	15
4. Mathematical Properties of the Elementary Solution and the Semigroup Associated with the Heat Operator	16
§3. Wave Equations	21
1. Model Problem: The Wave Equation in \mathbb{R}^n	21
2. The Euler–Poisson–Darboux Equation	44
3. An Application of §2 and 3: Viscoelasticity	48
§4. The Cauchy Problem for the Schrödinger Equation, Introduction	53
1. Model Problem 1. The Case of Zero Potential	53
2. Model Problem 2. The Case of a Harmonic Oscillator	57
§5. The Cauchy Problem for Evolution Equations Related to Convolution Products	58
1. Setting of Problem	58
2. The Method of the Fourier Transform	59
3. The Dirac Equation for a Free Particle	63
§6. An Abstract Cauchy Problem. Ovsyannikov's Theorem	66
Review of Chapter XIV	72

Chapter XV. Evolution Problems: The Method of Diagonalisation

Introduction	73
§1. The Fourier Method or the Method of Diagonalisation	74
1. The Case of the Space \mathbb{R}^1 ($n = 1$)	74
2. The Case of Space Dimension $n = 2$	94

3. The Case of Arbitrary Dimension n	99
Review	103
§2. Variations. The Method of Diagonalisation for an Operator Having Continuous Spectrum	104
1. Review of Self-Adjoint Operators in Hilbert Spaces	104
2. General Formulation of the Problem	104
3. A Simple Example of the Problem with Continuous Spectrum	108
§3. Examples of Application: The Diffusion Equation	112
1. Example of Application 1: The Monokinetic Diffusion Equation for Neutrons	112
2. Example of Application 2: The Age Equation in Problems of Slowing Down of Neutrons	118
3. Example of Application 3: Heat Conduction	122
§4. The Wave Equation: Mathematical Examples and Examples of Application	126
1. The Case of Dimension $n = 1$	126
2. The Case of Arbitrary Dimension n	143
3. Examples of Applications for $n = 1$	145
4. Examples of Applications for $n = 2$. Vibrating Membranes	156
5. Application to Elasticity; the Dynamics of Thin Homogeneous Beams	159
§5. The Schrödinger Equation	169
1. The Cauchy Problem for the Schrödinger Equation in a Domain $\Omega =]0, 1[\subset \mathbb{R}$	170
2. A Harmonic Oscillator	177
Review	183
§6. Application with an Operator Having a Continuous Spectrum: Example	184
Review of Chapter XV	186
Appendix. Return to the Problem of Vibrating Strings	186

Chapter XVI. Evolution Problems: The Method of the Laplace Transform

Introduction	202
§1. Laplace Transform of Distributions	203
1. Study of the Set I_f and Definition of the Laplace Transform	204
2. Properties of the Laplace Transform	210
3. Characterisation of Laplace Transforms of Distributions of $L_+(\mathbb{R})$	212
§2. Laplace Transform of Vector-valued Distributions	217
1. Distributions with Vector-valued Values	218
2. Fourier and Laplace Transforms of Vector-valued Distributions	222

§3. Applications to First Order Evolution Problems	225
1. ‘Vector-valued Distribution’ Solutions of an Evolution Equation of First Order in t	225
2. The Method of Transposition.	231
3. Application to First Order Evolution Equations. The Hilbert Space Case. L^2 Solutions in Hilbert Space.	233
4. The Case where A is Defined by a Sesquilinear Form $a(u, v)$	243
§4. Evolution Problems of Second Order in t	251
1. Direct Method	251
2. Use of Symbolic Calculus	257
Review	261
§5. Applications	261
1. Hydrodynamical Problems	261
2. A Problem of the Kinetics of Neutron Diffusion.	265
3. Problems of Diffusion of an Electromagnetic Wave	267
4. Problems of Wave Propagation.	273
5. Viscoelastic Problems	280
6. A Problem Related to the Schrödinger Equation	290
7. A Problem Related to Causality, Analyticity and Dispersion Relations	292
8. Remark 10	295
Review of Chapter XVI	296

Chapter XVII. Evolution Problems: The Method of Semigroups

Introduction	297
<i>Part A. Study of Semigroups</i>	301
§1. Definitions and Properties of Semigroups Acting in a Banach Space	301
1. Definition of a Semigroup of Class \mathcal{C}^0 (Resp. of a Group)	301
2. Basic Properties of Semigroups of Class \mathcal{C}^0	307
§2. The Infinitesimal Generator of a Semigroup	310
1. Examples	310
2. The Infinitesimal Generator of a Semigroup of Class \mathcal{C}^0	315
§3. The Hille–Yosida Theorem	321
1. A Necessary Condition	321
2. The Hille–Yosida Theorem	323
3. Examples of Application of the Hille–Yosida Theorem	327
§4. The Case of Groups of Class \mathcal{C}^0 and Stone’s Theorem	353
1. The Characterisation of the Infinitesimal Generator of a Group of Class \mathcal{C}^0	353
2. Unitary Groups of Class \mathcal{C}^0 . Stone’s Theorem	356

3. Applications of Stone's Theorem	357
4. Conservative Operators and Isometric Semigroups in Hilbert Space	362
Review	365
§5. Differentiable Semigroups	365
§6. Holomorphic Semigroups	367
§7. Compact Semigroups	388
1. Definition and Principal Properties	388
2. Characterisation of Compact Semigroups	389
3. Examples of Compact Semigroups	394
<i>Part B. Cauchy Problems and Semigroups</i>	397
§1. Cauchy Problems	397
§2. Asymptotic Behaviour of Solutions as $t \rightarrow +\infty$. Conservation and Dissipation in Evolution Equations	406
§3. Semigroups and Diffusion Problems	412
§4. Groups and Evolution Equations	420
1. Wave Problems	420
2. Schrödinger Type Problems	424
3. Weak Asymptotic Behaviour, for $t \rightarrow \pm \infty$, of Solutions of Wave Type of Schrödinger Type Problems	426
4. The Cauchy Problem for Maxwell's Equations in an Open Set $\Omega \subset \mathbb{R}^3$	433
§5. Evolution Operators in Quantum Physics. The Liouville-von Neumann Equation	439
1. Existence and Uniqueness of the Solution of the Cauchy Problem for the Liouville-von Neumann Equation in the Space of Trace Operators	439
2. The Evolution Equation of (Bounded) Observables in the Heisenberg Representation	446
3. Spectrum and Resolvent of the Operator h	451
§6. Trotter's Approximation Theorem	453
1. Convergence of Semigroups	453
2. General Representation Theorem	459
Summary of Chapter XVII	465

Chapter XVIII. Evolution Problems: Variational Methods

Introduction. Orientation	467
§1. Some Elements of Functional Analysis	469
1. Review of Vector-valued Distributions	469
2. The Space $W(a, b; V, V')$	472

3. The Spaces $W(a, b; X, Y)$	479
4. Extension to Banach Space Framework	482
5. An Intermediate Derivatives Theorem	493
6. Bidual. Reflexivity. Weak Convergence and Weak * Convergence	499
§2. Galerkin Approximation of a Hilbert Space	503
1. Definition	504
2. Examples	504
3. The Outline of a Galerkin Method	507
§3. Evolution Problems of First Order in t	509
1. Formulation of Problem (P)	509
2. Uniqueness of the Solution of Problem (P)	512
3. Existence of a Solution of Problem (P)	513
4. Continuity with Respect to the Data	520
5. Appendix: Various Extensions – Liftings	521
§4. Problems of First Order in t (Examples)	523
1. Mathematical Example 1. Dirichlet Boundary Conditions	524
2. Mathematical Example 2. Neumann Boundary Conditions	524
3. Mathematical Example 3. Mixed Dirichlet–Neumann Boundary Conditions	527
4. Mathematical Example 4. Bilinear Form Depending on Time t	528
5. Evolution, Positivity and ‘Maximum’ of Solutions of Diffusion Equations in $L^p(\Omega)$, $1 \leq p \leq \infty$	533
6. Mathematical Example 5. A Problem of Oblique Derivatives	539
7. Example of Application. The Neutron Diffusion Equation	542
8. A Stability Result	548
§5. Evolution Problems of Second Order in t	552
1. General Formulation of Problem (P_1)	552
2. Uniqueness in Problem (P_1)	558
3. Existence of a Solution of Problem (P_1)	561
4. Continuity with Respect to the Data	566
5. Formulation of Problem (P_2)	570
§6. Problems of Second Order in t. Examples	581
1. Mathematical Example 1	581
2. Mathematical Example 2	582
3. Mathematical Example 3	583
4. Mathematical Example 4	587
5. Application Examples	589
§7. Other Types of Equation	620
1. Schrödinger Type Equations	620
2. Evolution Equations with Delay	643
3. Some Integro-Differential Equations	651
4. Optimal Control and Problems where the Unknowns are Operators	662

5. The Problem of Coupled Parabolic-Hyperbolic Transmission	670
6. The Method of 'Extension with Respect to a Parameter'.	676
Review of Chapter XVIII	679
Bibliography	680
Table of Notations	686
Index	702
Contents of Volumes 1–4, 6	705