

# Contents

## Preface      vii

<b>1</b>	<b>Introduction</b>	<b>1</b>
1.1	The special role of logic in rational inquiry	1
1.2	Why learn an artificial language?	2
1.3	About this book	4

## Part I Propositional Logic      7

<b>2</b>	<b>Atomic Sentences</b>	<b>9</b>
2.1	Individual constants	9
2.2	Predicate symbols	10
2.3	Atomic sentences	12
2.4	The first-order language of set theory	15
2.5	Function symbols	17
2.6	The first-order language of arithmetic	19
2.7	General first-order languages	20
2.8	Methods of proof	24
2.9	Formal proofs	30
2.10	Alternative notation	33
<b>3</b>	<b>Conjunctions, Disjunctions, and Negations</b>	<b>35</b>
3.1	Negation symbol ( $\neg$ )	36
3.2	Conjunction symbol ( $\wedge$ )	37
3.3	Disjunction symbol ( $\vee$ )	39
3.4	Ambiguity and parentheses	42
3.5	Logical equivalence	44
3.6	Translation	47
3.7	Satisfiability and logical truth	51
3.8	Methods of proof involving $\neg$ , $\wedge$ , and $\vee$	58
3.9	Formal proofs	66
3.10	Conjunctive and disjunctive normal forms	82

3.11	Truth-functional completeness	85
3.12	Alternative notation	88
<b>4</b>	<b>Conditionals and Biconditionals</b>	<b>91</b>
4.1	Material conditional symbol ( $\rightarrow$ )	91
4.2	Biconditional symbol ( $\leftrightarrow$ )	93
4.3	Conversational implicature	97
4.4	Methods of proof involving $\rightarrow$ and $\leftrightarrow$	99
4.5	Formal proofs	104
4.6	$\mathcal{F}'$ : Speeding up system $\mathcal{F}$	108
4.7	Alternative notation	112
<b>Part II Quantifiers</b>		<b>113</b>
<b>5</b>	<b>Introduction to Quantification</b>	<b>115</b>
5.1	Variables	115
5.2	Atomic wffs	115
5.3	Quantifiers	116
5.4	Wffs and sentences	117
5.5	Semantics for the quantifiers	119
5.6	Game rules for the quantifiers	121
5.7	The four Aristotelian forms	123
5.8	Translating complex noun phrases	125
5.9	Logical equivalences involving negation and quantifiers	132
5.10	Methods of proof involving $\forall$ and $\exists$	135
5.11	Formal proofs	144
5.12	Function symbols, revisited	150
5.13	Alternative notation	152
<b>6</b>	<b>Sentences with More than One Quantifier</b>	<b>154</b>
6.1	Multiple uses of a single quantifier symbol	154
6.2	Mixed quantifiers	157
6.3	The step-by-step method of translation	160
6.4	Paraphrasing English	162
6.5	Ambiguity and context sensitivity	167
6.6	Translations using function symbols	170
6.7	Methods of proof involving mixed quantifiers	172
6.8	Formal proofs	177
6.9	Prenex form	181
<b>7</b>	<b>Some Specific Uses of Quantifiers</b>	<b>185</b>
7.1	Making numerical claims	185
7.2	Definite descriptions	189
7.3	Methods of proof involving numerical claims	190

- 7.4 Some review problems 194
- 7.5 Expressive limitations of first-order logic 201

## **Part III Applications of First-order Logic 205**

- 8 First-order Set Theory 207**
  - 8.1 Cantor's set theory 208
  - 8.2 Singletons and the empty set 211
  - 8.3 The subset relation 212
  - 8.4 Intersection and union 214
  - 8.5 Sets of sets 216
  - 8.6 The powerset of a set 218
  - 8.7 Russell's Paradox 220
  - 8.8 Modern set theory 222
- 9 Induction 227**
  - 9.1 Some examples of induction 228
  - 9.2 Inductive definitions in set theory 234
  - 9.3 Induction on the natural numbers 235

## **Part IV Advanced Topics 241**

- 10 Advanced Topics in Propositional Logic 243**
  - 10.1 Horn sentences 243
  - 10.2 Satisfiable sentences and  
logically true sentences, revisited 247
  - 10.3 Resolution 250
  - 10.4 The conditional form of Horn sentences 255
- 11 Advanced Topics in FOL 258**
  - 11.1 First-order structures 258
  - 11.2 Spurious structures 262
  - 11.3 Truth and satisfaction, revisited 263
  - 11.4 Skolemization 269
  - 11.5 Unification of terms 270
  - 11.6 Resolution, revisited 273
  - 11.7 Completeness and incompleteness 278

## **Reference Material**

- A How to Use Tarski's World 283**
  - A.1 The basics 283
    - Starting Tarski's World 283
    - The four main windows 285
    - Opening saved files 285

	Starting new files	287
	Closing files and windows	287
	Saving a file	287
	Naming files	288
	Saving a file on another disk	288
	Exiting	289
A.2	Using the world window	289
	Adding blocks	289
	Naming blocks	289
	Moving blocks	290
	Sizing and shaping blocks	290
	Deleting blocks	290
	Hiding labels	290
	2-D view	291
	Rotating worlds	291
A.3	Using the keyboard and sentence windows	291
	Writing formulas	291
	Commenting your sentences	292
	Creating a list of sentences	292
	Moving from sentence to sentence	293
	Deleting sentences	293
	Editing sentences	294
	Cutting, copying, and pasting	295
	Moving and sizing windows	295
	Printing	296
A.4	Using the sentence inspector	296
	Checking syntax	296
	Verifying truth	297
A.5	Playing the game	298
	Picking blocks and sentences	298
	Backing up and giving up	299
	When to play the game	299
A.6	Summary of Keyboard Equivalents	300
<b>B</b>	<b>Windows Terminology</b>	<b>302</b>
<b>C</b>	<b>Summary of Proof Rules</b>	<b>305</b>
	C.1 System $\mathcal{F}$	305
	C.2 System $\mathcal{F}'$	307
	<b>General Index</b>	<b>310</b>
	<b>Tarski's World Index</b>	<b>317</b>
	<b>Exercise Files Index</b>	<b>318</b>