

CONTENTS OF VOLUME II

Constructivism in Mathematics: Contents

Preliminaries	xiii
7. The topology of metric spaces	345
1. Basic definitions	345
2. Complete, separable metric spaces	350
3. Located sets	358
4. Complete, totally bounded spaces	363
5. Locally compact spaces	371
6. Notes	376
Exercises	379
8. Algebra	383
1. Identity, apartness and order	383
2. Groups	388
3. Rings and modules	399
4. Linear algebra	407
5. Polynomial rings	415
6. Fields and local rings	430
7. The fundamental theorem of algebra	434
8. Notes	438
Exercises	439
9. Finite-type arithmetic and theories of operators	443
1. Intuitionistic finite-type arithmetic	444
2. Normalization, and a term model for \mathbf{HA}^ω	458
3. The theory \mathbf{APP}	472
4. Models for \mathbf{APP}	480
5. Abstract realizability in \mathbf{APP}	491
6. Extensionality and choice in \mathbf{APP} and \mathbf{HA}^ω	495

7. Some metamathematical applications	501
8. Theories of operators and classes	512
9. Notes	517
Exercises	520
10. Proof theory of intuitionistic logic	527
1. Preliminaries	527
2. Normalization	532
3. The structure of normal derivations of N-IQCE	538
4. The decidability of IPC	541
5. Other applications of normalization	544
6. Conservative addition of predicative classes	548
7. Sequent calculi	550
8. N-IQC as a calculus of terms	556
9. Notes	563
Exercises	565
11. The theory of types and constructive set theory	571
1. Towards a theory of types	571
2. The theory \mathbf{ML}_0^i	575
3. Some alternative formulations of \mathbf{ML}_0^i	589
4. The types N_k and reformulation of the E-rules	592
5. The theory \mathbf{ML}_0	603
6. Embeddings into APP	608
7. Extensions of \mathbf{ML}_0^i and \mathbf{ML}_0	611
8. Constructive set theory	619
9. Notes	632
Exercises	635
12. Choice sequences	639
1. Introduction	639
2. Lawless sequences	645
3. The elimination translation for the theory LS	658
4. Other notions of choice sequence	665
5. Notes	672
Exercises	674
13. Semantical completeness	677
1. Beth models	677
2. Completeness for intuitionistic validity	685

3. Incompleteness results	694
4. Lattices, Heyting algebras and complete Heyting algebras	700
5. Algebraic semantics for IPC	705
6. Ω -sets and structures	709
7. Validity as forcing	719
8. Postscript on realizability	724
9. Notes	729
Exercises	733
14. Sheaves, sites and higher-order logic	737
1. Presheaves, sheaves and sheaf-completion	738
2. Ω -presheaf and Ω -sheaf structures	747
3. Some notions from category theory	751
4. Forcing over sites	759
5. Sheaf models for higher-order logic	767
6. Notes	775
Exercises	777
15. Applications of sheaf models	781
1. Interpretation of \mathbb{N} , \mathbb{Q} , \mathbb{Z} , \mathbb{R} , $\mathbb{N}^{\mathbb{N}}$ in $\text{Sh}(\mathcal{O}(T))$	782
2. The axiom of countable choice	786
3. Topologies in sheaves over a cHa	789
4. A derived rule of local continuity	805
5. The monoid model for CS	808
6. A site model for LS	820
7. Notes	825
Exercises	827
16. Epilogue	831
1. The role of language and “informal rigour”	831
2. Intuitionistic logic, formalisms, and equality	835
3. Brouwer’s theory of the creative subject	842
4. Dummett’s anti-realist argument	846
Bibliography	853
Index	I
Index of names	XXVII
List of symbols	XXXVII