

# Contents

<b>Preface</b>	<b>vii</b>
<b>Acknowledgments</b>	<b>ix</b>
<b>Illustration Credits</b>	<b>xi</b>
<b>Chapter 1. Introduction</b>	<b>1</b>
1.1. A Brief History of Space-Filling Curves	1
1.2. Notation	2
1.3. Definitions and Netto's Theorem	4
1.4. Problems	6
<b>Chapter 2. Hilbert's Space-Filling Curve</b>	<b>9</b>
2.1. Generation of Hilbert's Space-Filling Curve	9
2.2. Nowhere Differentiability of the Hilbert Curve	12
2.3. A Complex Representation of the Hilbert Curve	13
2.4. Arithmetization of the Hilbert Curve	18
2.5. An Analytic Proof of the Nowhere Differentiability of the Hilbert Curve	19
2.6. Approximating Polygons for the Hilbert Curve	21
2.7. Moore's Version of the Hilbert Curve	24
2.8. A Three-Dimensional Hilbert Curve	26
2.9. Problems	29
<b>Chapter 3. Peano's Space-Filling Curve</b>	<b>31</b>
3.1. Definition of Peano's Space-Filling Curve	31
3.2. Nowhere Differentiability of the Peano Curve	34
3.3. Geometric Generation of the Peano Curve	34
3.4. Proof that the Peano Curve and the Geometric Peano Curve are the Same	36
3.5. Cesàro's Representation of the Peano Curve	40
3.6. Approximating Polygons for the Peano Curve	42
3.7. Wunderlich's Versions of the Peano Curve	43

3.8.	A Three-Dimensional Peano Curve	45
3.9.	Problems	46
<b>Chapter 4.</b>	<b>Sierpiński's Space-Filling Curve</b>	<b>49</b>
4.1.	Sierpiński's Original Definition	49
4.2.	Geometric Generation and Knopp's Representation of the Sierpiński Curve	51
4.3.	Representation of the Sierpiński-Knopp Curve in Terms of Quaternaries	56
4.4.	Nowhere Differentiability of the Sierpiński- Knopp Curve	58
4.5.	Approximating Polygons for the Sierpiński- Knopp Curve	60
4.6.	Pólya's Generalization of the Sierpiński-Knopp Curve	62
4.7.	Problems	67
<b>Chapter 5.</b>	<b>Lebesgue's Space-Filling Curve</b>	<b>69</b>
5.1.	The Cantor Set	69
5.2.	Properties of the Cantor Set	71
5.3.	The Cantor Function and the Devil's Staircase	74
5.4.	Lebesgue's Definition of a Space-Filling Curve	76
5.5.	Approximating Polygons for the Lebesgue Curve	79
5.6.	Problems	82
<b>Chapter 6.</b>	<b>Continuous Images of a Line Segment</b>	<b>85</b>
6.1.	Preliminary Remarks and a Global Characterization of Continuity	85
6.2.	Compact Sets	91
6.3.	Connected Sets	94
6.4.	Proof of Netto's Theorem	97
6.5.	Locally Connected Sets	98
6.6.	A Theorem by Hausdorff	99
6.7.	Pathwise Connectedness	101
6.8.	The Hahn-Mazurkiewicz Theorem	106
6.9.	Generation of Space-Filling Curves by Stochastically Independent Functions	108
6.10.	Representation of a Space-Filling Curve by an Analytic Function	112
6.11.	Problems	115
<b>Chapter 7.</b>	<b>Schoenberg's Space-Filling Curve</b>	<b>119</b>
7.1.	Definition and Basic Properties	119
7.2.	The Nowhere Differentiability of the Schoenberg Curve	121

7.3. Approximating Polygons	123
7.4. A Three-Dimensional Schoenberg Curve	127
7.5. An $\aleph_0$ -Dimensional Schoenberg Curve	128
7.6. Problems	129
<b>Chapter 8. Jordan Curves of Positive Lebesgue Measure</b>	<b>131</b>
8.1. Jordan Curves	131
8.2. Osgood's Jordan Curves of Positive Measure	132
8.3. The Osgood Curves of Sierpiński and Knopp	136
8.4. Other Osgood Curves	140
8.5. Problems	142
<b>Chapter 9. Fractals</b>	<b>145</b>
9.1. Examples	145
9.2. The Space where Fractals are Made	149
9.3. The Invariant Attractor Set	154
9.4. Similarity Dimension	156
9.5. Cantor Curves	159
9.6. The Heighway-Dragon	162
9.7. Problems	165
<b>Appendix</b>	<b>169</b>
A.1. Computer Programs	169
A.1.1. Computation of the Nodal Points of the Hilbert Curve	169
A.1.2. Computation of the Nodal Points of the Peano Curve	170
A.1.3. Computation of the Nodal Points of the Sierpiński-Knopp Curve	171
A.1.4. Plotting Program for the Approximating Polygons of the Schoenberg Curve	172
A.2. Theorems from Analysis	173
A.2.1. Binary and Other Representations	173
A.2.2. Condition for Non-Differentiability	174
A.2.3. Completeness of the Euclidean Space	174
A.2.4. Uniform Convergence	174
A.2.5. Measure of the Intersection of a Decreasing Sequence of Sets	174
A.2.6. Cantor's Intersection Theorem	175
A.2.7. Infinite Products	175
<b>References</b>	<b>177</b>
<b>Index</b>	<b>187</b>