

# Contents

<b>1</b>	<b>Introduction</b>	<b>1</b>
1.1	Functions	1
1.2	Formal Systems	3
1.3	Mathematical Induction	4
1.4	Summary	5
<b>2</b>	<b>Notation and the Basic Theory</b>	<b>7</b>
2.1	Notation	7
2.2	The theory $\lambda$	11
2.3	Substitution	15
2.3.1	Three Approaches	15
2.3.2	The Substitution Lemma	19
2.4	Extensionality	21
2.5	Consistency and Completeness	22
2.6	Summary	25
<b>3</b>	<b>Reduction</b>	<b>27</b>
3.1	Introduction	27
3.2	Notions of Reduction	29
3.3	The Church–Rosser Theorem	34
3.4	Delta Rules	41
3.5	Residuals	45
3.6	Head Normal Forms	47
3.7	The Standardisation Theorem	49
3.8	Summary	50
<b>4</b>	<b>Combinatory Logic</b>	<b>51</b>
4.1	Combinatory Logic	51
4.2	Combinatory Logic and the $\lambda$ -calculus	58
4.3	Bases	61
4.4	Summary	62
<b>5</b>	<b>Semantics</b>	<b>63</b>
5.1	Models	63
5.1.1	$\lambda$ -algebras	65
5.1.2	$\lambda$ -models	71
5.1.3	Term models	72
5.2	Böhm Trees	73
5.2.1	Böhm-like Trees	73

5.2.2	The Model $B$	75
5.3	Summary	77
<b>6</b>	<b>Computability</b>	<b>79</b>
6.1	Fixed Points	79
6.2	Numeral Systems	83
6.3	$\lambda$ -definability	87
6.4	Decidability	90
6.5	Summary	93
<b>7</b>	<b>Types</b>	<b>95</b>
7.1	Typed $\lambda$ -calculus	95
7.2	The Polymorphic $\lambda$ -calculus	103
7.3	Intersection Types	105
7.4	Summary	107
<b>8</b>	<b>Practical Issues</b>	<b>109</b>
8.1	Reduction machines	110
8.1.1	Krivine's Machine	112
8.1.2	An Eager Machine	113
8.1.3	Correctness	114
8.2	Needed Reductions	115
8.3	Strictness Analysis	120
8.4	Polymorphic Type Inference	126
8.5	Summary	134
<b>9</b>	<b>Other Calculi</b>	<b>135</b>
9.1	The Lazy $\lambda$ -calculus	135
9.1.1	The theory of the lazy $\lambda$ -calculus	136
9.2	The $\gamma$ -calculus	140
9.2.1	The theory of the $\gamma$ -calculus	140
9.2.2	Relating the $\gamma$ -calculus to the $\lambda$ -calculus	145
9.3	The $\lambda\sigma$ -calculus	146
9.3.1	The basic theory of the $\lambda\sigma$ -calculus	147
9.3.2	Relating the $\lambda\sigma$ -calculus to the $\lambda$ -calculus	148
9.3.3	Towards an abstract machine	149
9.4	Summary	151
<b>10</b>	<b>Further Reading</b>	<b>153</b>
10.1	General	153
10.2	Reduction	153
10.3	Combinatory Logic	153
10.4	Semantics	154
10.5	Computability	154
10.6	Types	154

10.7 Practical Issues	154
10.8 Other Calculi	154
10.9 Summary	154
<b>Bibliography</b>	<b>157</b>
<b>Index</b>	<b>159</b>