Contents

Introduction xi

CHAPTER 1. Divergence form elliptic equations 1
§1. Preliminaries 1
§2. The classical Dirichlet problem 5
§3. Estimates for harmonic measure 8
§4. Fatou type theorems 13
§5. Area integrals, BMO ($d\omega$) and Hardy spaces 19
§6. The Neumann problem: variational and weak formulations 26
§7. The Dirichlet, Neumann and Regularity problems with L^p data. Formulations of the problems 28
§8. Some general consequences of $(R)_{q_0}$ and $(N)_{q_0}$ 31
§9. Counterexamples based on the Beurling-Ahlfors theorem for quasi-conformal mappings 38
§10. Some approximation results 40
§11. Notes 42

CHAPTER 2. Some classes of examples and their perturbation theory 45
§1. The Dirichlet, Neumann and Regularity problems for the Laplacian on Lipschitz and C^1 domains 45
§2. The method of layer potentials for Laplace's equation on Lipschitz and C^1 domains 50
§3. Hardy spaces of harmonic functions on Lipschitz domains 57
§4. The Dirichlet, Neumann and Regularity problems for operators with radially independent coefficients 63
§5. The multilinear singular integral approach to the radially independent case and its perturbation theory. Extensions to the complex coefficient case and its connections with the boundedness of the Cauchy integral and Kato's square root problem 67
§6. Some analogies between the perturbation theory for the Dirichlet problem and classical differentiation theory 75
§7. Perturbation theory for the Dirichlet problem 83
§8. Perturbation theory for the regularity and Neumann problems 97
§9. Some examples related to the relationship between $(R)_p$, $(N)_p$ and $(D)_{p'}$ 108
§10. Notes 111

CHAPTER 3. Epilogue: Some further results and open problems 114
§1. General divergence form equations 114
§2. Other results and open problems for constant coefficient operators on domains 115
§3. Other results and open problems for variable coefficient equations 127

References 134