

Contents

PART I PROLEGOMENA TO STABILITY THEORY

1 General introduction	3
2 Review of stability theory	19
2.1 Introduction	19
2.2 Notation: germs, jets, equivalence relations	20
2.3 C^∞ -stability	25
2.4 Finite singularity type	29
2.5 Stratification theory	32
2.6 C^0 -stability	43
3 Proper maps and function space topologies	49
3.1 Introduction	49
3.2 Topology of proper maps	50
3.3 Two useful lemmas	57
3.4 Function space topologies	58
3.5 Existence of global unfoldings	72
3.6 Unfoldings and stability	77
3.7 Proofs of stability theorems	84
4 Various notions of stability	92
4.1 Introduction	92
4.2 Global stability: compact source	95
4.3 Global stability: non-compact source	100
4.4 Local and semi-global stability	111
4.5 The relation between global and local stability	114

PART II NECESSARY CONDITIONS

5 Disruptive germ classes	121
5.1 Introduction	121
5.2 Germ classes	122
5.3 Local definitions	124
5.4 Persistence	130
5.5 Disruptive germs	136
5.6 Strong disruptiveness	140
5.7 Disruptiveness and instability	148
5.8 Quasi-disruptive germs	152
5.9 Finite C^0 -determinacy	158
6 Necessary conditions for topological stability	160
6.1 Introduction	160
6.2 Direct application of results from Chapter 5	162
6.3 The immersion condition	169
6.4 Inductive criteria for ST-invariance and disruptiveness	177
6.5 Quasi-disruptiveness	182
6.6 Applications of quasi-disruptiveness	189
6.7 LST-invariants and regularity conditions	193
7 Stable topological invariance of μ-constant strata	199
7.1 Introduction	199
7.2 Simple and unimodal singularities	201
7.3 Exceptional deformations	209
7.4 Triple and quadruple points	215
7.5 Functions of three variables with reduced 3-jet	220
7.6 Complete intersection curves	226
7.7 Complete intersection cusp singularities	240
7.8 Complete intersection surfaces	251
8 Stable topological type of finite map-germs	260
8.1 Introduction	260
8.2 Codimensions	263
8.3 Σ^2 : first cases	268
8.4 $\Sigma^{2,1}$ germs	273
8.5 $\Sigma^{2,2}$ germs of order 3	283
8.6 Germs of $\Sigma^{n(2)}$ type	291
8.7 Germs of $\Sigma^{3(3)}$ type	314
Appendix: table of equidimensional strata	339

PART III SUFFICIENT CONDITIONS

9 A sufficient condition for strong topological stability	345
9.1 Introduction	345
9.2 From local to global stability	350
9.3 Tame retractions	357
9.4 Multitransversality implies tame $P\text{-}C^0$ -stability	376
9.5 Constructing global tame retractions from local ones	379
9.6 Civilization and weighted homogeneity	386
10 Algebraic criteria for civilization	398
10.1 Introduction	398
10.2 Instability loci	399
10.2.1 Standard notations	399
10.2.2 Recapitulation of Theorem 9.6.6	401
10.2.3 Example: $J(3, 4)$	403
10.2.4 Example: the H -series	404
10.2.5 Example: $LFV_{3,3}$	406
10.2.6 Example: $LFT_{3,3}$ ($e = 1$)	408
10.3 A criterion for civilization	410
10.3.1 Geometrical discussion	410
10.3.2 An algebraic criterion for civilization	412
10.3.3 Transversality properties of almost-stable mappings	416
10.4 Instability ideals and discriminant matrices	421
10.4.1 Instability ideals and discriminant matrices	421
10.4.2 An algorithm for discriminant matrices	424
10.4.3 The 1-jet of the discriminant matrix and the basic pairing	427
10.4.4 Example: $\Sigma^{4(2)}$ (generic stratum)	431
10.4.5 Example: the stratum $FA_3 \cup FC_0$ ($s = t$)	433
10.4.6 Refinements of the algorithm	435
10.4.7 Example: $LFT_{3,3}$ revisited	439
10.4.8 Comparison with Damon's algorithm	443
10.5 Further properties of discriminant matrices	445
10.5.1 Consequences of finite singularity type	445
10.5.2 Examples: \tilde{D}_{n+1}	448
10.5.3 Refinements of the discriminant matrix	451
10.5.4 Poincaré series	460
10.5.5 Example: the J and F series	465
10.5.6 Symmetry of the discriminant matrix and the E -series	472

10.6	Extensions	479
10.6.1	General remarks	479
10.6.2	Simplifications of the theory	483
10.6.3	Examples: extensions of the $D_{l,l+2}$ and $I(2,l)$	485
10.6.4	Applications to J -extensions and 0-extensions	488
10.6.5	Extensions of $J(3,4)$	491
10.6.6	Extensions of $LFV_{3,3}$	492
11	Calculations of instability loci	494
11.1	Description of methods of calculation	494
11.2	Interpretation of the data	501
11.3	Summary of calculations: hypersurface cases $s > t = 1$	503
11.4	Summary of calculations: complete intersections with $s > t > 1$	515
11.5	Summary of calculations: the equidimensional case $s = t$	528
11.6	Summary of calculations: cases $s < t$	534
PART IV CONCLUSION		
12	Necessary and sufficient conditions for stability	543
12.1	Introduction	543
12.2	Stability and multitransversality	543
12.3	The case $n > p$	546
12.4	The case $n \leq p$	549
12.5	Necessary and sufficient conditions for stability	552
Problems 554		
References 557		
Index of terminology 568		