

# Contents

|  |            |
|--|------------|
| <b>Preface to the Second Edition</b>           | <b>iii</b> |
| <b>Preface to the First Edition</b>            | <b>v</b>   |
| <br>   |            |
| <b>Chapter 1. Some Preliminaries</b>           | <b>1</b>   |
| 1. The Rudiments of Set Theory                 | 1          |
| 2. Some Logic                                  | 6          |
| 3. Mathematical Induction                      | 9          |
| 4. Inequalities and Absolute Value             | 13         |
| 5. Equivalence Relations                       | 16         |
| <br>   |            |
| <b>Chapter 2. Vector Spaces</b>                | <b>19</b>  |
| 1. The Cartesian Plane                         | 19         |
| 2. The Definition of a Vector Space            | 24         |
| 3. Some Elementary Properties of Vector Spaces | 30         |
| 4. Subspaces                                   | 33         |
| 5. Linear Transformations                      | 39         |
| 6. Linear Transformations on Euclidean Spaces  | 46         |
|  | <b>vii</b> |

|  |            |
|--|------------|
| <b>Chapter 3. The Derivative</b>   | <b>55</b>  |
| 1. Normed Vector Spaces  | 56         |
| 2. Open and Closed Sets  | 60         |
| 3. Continuous Functions Between Normed Vector Spaces                       | 65         |
| 4. Elementary Properties of Continuous Functions                           | 75         |
| 5. The Derivative  | 80         |
| 6. Elementary Properties of the Derivative                                 | 85         |
| 7. Partial Derivatives and the Jacobian Matrix                             | 87         |
| <br>   |            |
| <b>Chapter 4. The Structure of Vector Spaces</b>                           | <b>95</b>  |
| 1. Spans and Linear Independence   | 96         |
| 2. Bases   | 101        |
| 3. Bases and Linear Transformations  | 105        |
| 4. The Dimension of a Vector Space   | 110        |
| 5. Inner Product Spaces  | 116        |
| 6. The Norm on an Inner Product Space                                      | 121        |
| 7. Orthonormal Bases   | 123        |
| 8. The Cross Product in $\mathbb{R}^3$                                     | 130        |
| <br>   |            |
| <b>Chapter 5. Compact and Connected Sets</b>                               | <b>133</b> |
| 1. Convergent Sequences  | 133        |
| 2. Compact Sets  | 139        |
| 3. Upper and Lower Bounds  | 143        |
| 4. Continuous Functions on Compact Sets                                    | 147        |
| 5. A Characterization of Compact Sets                                      | 151        |
| 6. Uniform Continuity  | 155        |
| 7. Connected Sets  | 157        |
| <br>   |            |
| <b>Chapter 6. The Chain Rule, Higher Derivatives, and Taylor's Theorem</b> | <b>163</b> |
| 1. The Chain Rule  | 164        |
| 2. Proof of the Chain Rule   | 170        |
| 3. Higher Derivatives  | 173        |
| 4. Taylor's Theorem for Functions of One Variable                          | 181        |
| 5. Taylor's Theorem for Functions of Two Variables                         | 186        |
| 6. Taylor's Theorem for Functions of $n$ Variables                         | 191        |
| 7. A Sufficient Condition for Differentiability                            | 195        |
| 8. The Equality of Mixed Partial Derivatives                               | 200        |

|  |            |
|--|------------|
| <b>Chapter 7. Linear Transformations and Matrices</b>        | <b>203</b> |
| 1. The Matrix of a Linear Transformation                     | 203        |
| 2. Isomorphisms and Invertible Matrices                      | 207        |
| 3. Change of Basis   | 211        |
| 4. The Rank of a Matrix                                      | 216        |
| 5. The Trace and Adjoint of a Linear Transformation          | 219        |
| 6. Row and Column Operations                                 | 227        |
| 7. Gaussian Elimination                                      | 232        |
| <b>Chapter 8. Maxima and Minima</b>                          | <b>239</b> |
| 1. Maxima and Minima at Interior Points                      | 240        |
| 2. Quadratic Forms   | 246        |
| 3. Criteria for Local Maxima and Minima                      | 252        |
| 4. Constrained Maxima and Minima: I                          | 257        |
| 5. The Method of Lagrange Multipliers                        | 263        |
| 6. Constrained Maxima and Minima: II                         | 268        |
| 7. The Proof of Proposition 2.3                              | 273        |
| <b>Chapter 9. The Inverse and Implicit Function Theorems</b> | <b>277</b> |
| 1. The Inverse Function Theorem                              | 277        |
| 2. The Proof of Theorem 1.3                                  | 283        |
| 3. The Proof of the General Inverse Function Theorem         | 285        |
| 4. The Implicit Function Theorem: I                          | 292        |
| 5. The Implicit Function Theorem: II                         | 296        |
| <b>Chapter 10. The Spectral Theorem</b>                      | <b>303</b> |
| 1. Complex Numbers   | 304        |
| 2. Complex Vector Spaces                                     | 308        |
| 3. Eigenvectors and Eigenvalues                              | 314        |
| 4. The Spectral Theorem                                      | 318        |
| 5. Determinants  | 324        |
| 6. Properties of the Determinant                             | 329        |
| 7. More on Determinants                                      | 336        |
| 8. Quadratic Forms   | 340        |
| <b>Chapter 11. Integration</b>                               | <b>345</b> |
| 1. Integration of Functions of One Variable                  | 345        |
| 2. Properties of the Integral                                | 353        |
| 3. The Integral of a Function of Two Variables               | 359        |
| 4. The Integral of a Function of $n$ Variables               | 366        |

|  |            |
|--|------------|
| 5. Properties of the Integral                                | 375        |
| 6. Integrable Functions                                      | 379        |
| 7. The Proof of Theorem 6.2                                  | 384        |
| <b>Chapter 12. Iterated Integrals and the Fubini Theorem</b> | <b>387</b> |
| 1. The Fubini Theorem  | 387        |
| 2. Integrals Over Nonrectangular Regions                     | 393        |
| 3. More Examples   | 399        |
| 4. The Proof of Fubini's Theorem                             | 407        |
| 5. Differentiating Under the Integral Sign                   | 409        |
| 6. The Change of Variable Formula                            | 412        |
| 7. The Proof of Theorem 6.2                                  | 418        |
| <b>Chapter 13. Line Integrals</b>                            | <b>427</b> |
| 1. Curves  | 427        |
| 2. Line Integrals of Functions                               | 431        |
| 3. Line Integrals of Vector Fields                           | 436        |
| 4. Conservative Vector Fields                                | 443        |
| 5. Green's Theorem   | 450        |
| 6. The Proof of Green's Theorem                              | 455        |
| <b>Chapter 14. Surface Integrals</b>                         | <b>461</b> |
| 1. Surfaces  | 461        |
| 2. Surface Area  | 468        |
| 3. Surface Integrals   | 475        |
| 4. Stokes' Theorem   | 481        |
| <b>Chapter 15. Differential Forms</b>                        | <b>487</b> |
| 1. The Algebra of Differential Forms                         | 487        |
| 2. Basic Properties of the Sum and Product of Forms          | 492        |
| 3. The Exterior Differential                                 | 494        |
| 4. Basic Properties of the Exterior Differential             | 500        |
| 5. The Action of Differentiable Functions on Forms           | 502        |
| 6. Further Properties of the Induced Mapping                 | 505        |
| <b>Chapter 16. Integration of Differential Forms</b>         | <b>509</b> |
| 1. Integration of Forms                                      | 509        |
| 2. The General Stokes' Theorem                               | 513        |
| 3. Green's Theorem and Stokes' Theorem                       | 517        |
| 4. The Gauss Theorem and Incompressible Fluids               | 519        |
| 5. Proof of the General Stokes' Theorem                      | 525        |

|  |            |
|--|------------|
| <b>Contents</b>                                  | <b>xi</b>  |
| <b>Appendix 1. The Existence of Determinants</b> | <b>527</b> |
| <b>Appendix 2. Jordan Canonical Form</b>         | <b>531</b> |
| 1. Generalized Eigenvalues                       | 531        |
| 2. The Jordan Canonical Form                     | 534        |
| 3. Polynomials and Linear Transformations        | 540        |
| 4. The Proof of Theorem 3.5                      | 545        |
| 5. The Proof of Theorem 2.2                      | 549        |
| <b>Solutions of Selected Exercises</b>           | <b>553</b> |
| <b>Index</b>                                     | <b>577</b> |