

Contents

1. Foundational Material	1
1.1 Manifolds and Differentiable Manifolds	1
1.2 Tangent Spaces	5
1.3 Submanifolds	9
1.4 Riemannian Metrics	12
1.5 Vector Bundles	32
1.6 Integral Curves of Vector Fields. Lie Algebras	40
1.7 Lie Groups	49
1.8 Exercises for Chapter 1	53
2. De Rham Cohomology and Harmonic Differential Forms	55
2.1 The Laplace Operator	55
2.2 Representing Cohomology Classes by Harmonic Forms	63
2.3 Generalizations	72
2.4 Exercises for Chapter 2	73
3. Parallel Transport, Connections, and Covariant Derivatives	77
3.1 Connections in Vector Bundles	77
3.2 Metric Connections. The Yang-Mills Functional	86
3.3 The Levi-Civita Connection	95
3.4 The Geometry of Submanifolds. Minimal Submanifolds	108
3.5 The Bochner Method	120
3.6 Exercises for Chapter 3	123
4. Geodesics and Jacobi Fields	125
4.1 1st and 2nd Variation of Arclength and Energy	125
4.2 Jacobi Fields	131
4.3 Conjugate Points and Distance Minimizing Geodesics	140
4.4 Riemannian Manifolds of Constant Curvature	148
4.5 The Rauch Comparison Theorems and other Jacobi Field Estimates	150
4.6 Geometric Applications of Jacobi Field Estimates	155

4.7	Approximate Fundamental Solutions and Representation Formulae	160
4.8	Exercises for Chapter 4	162
A Short Survey on Curvature and Topology		165
5.	Morse Theory and Closed Geodesics	173
5.1	Preparations	173
5.2	Critical Points of Functions and the Topology of Manifolds	174
5.3	The Morse Inequalities (Including an Introduction to Algebraic Topology)	181
5.4	Spaces of Curves in Riemannian Manifolds	198
5.5	The Theorem of Lyusternik and Fet	205
5.6	Exercises for Chapter 5	209
6.	Symmetric Spaces and Kähler Manifolds	211
6.1	Complex Projective Space. Definition of Kähler Manifolds	211
6.2	The Geometry of Symmetric Spaces	221
6.3	Some Results about the Structure of Symmetric Spaces	231
6.4	The Space $Sl(n, \mathbb{R})/SO(n, \mathbb{R})$	237
6.5	Symmetric Spaces of Noncompact Type as Examples of Nonpositively Curved Riemannian Manifolds	255
6.6	Exercises for Chapter 6	261
7.	The Palais-Smale Condition and Closed Geodesics	263
7.1	The Palais-Smale Condition	263
7.2	The Palais-Smale Condition for Closed Geodesics	265
7.3	Exercises for Chapter 7	275
8.	Harmonic Maps	277
8.1	Definitions	277
8.2	Two-dimensional Harmonic Mappings and Holomorphic Quadratic Differentials	282
8.3	The Existence of Harmonic Maps in Two Dimensions	295
8.4	Definition and Lower Semicontinuity of the Energy Integral. Regularity Questions for Weakly Harmonic Maps and Weak Minimal Surfaces	318
8.5	Higher Regularity	338
8.6	Formulae for Harmonic Maps. The Bochner Technique	350
8.7	Harmonic Maps into Manifolds of Nonpositive Curvature	360
8.8	Exercises for Chapter 8	382

Appendix A: Linear Elliptic Partial Differential Equations ..	385
A.1 Sobolev Spaces	385
A.2 Existence and Regularity Theory for Solutions of Linear Elliptic Equations	388
Appendix B: Fundamental Groups and Covering Spaces	393
Index	397