

Contents

Foreword by G.E. Sacks	vii
Preface	ix
Introduction	1
What is ‘Classical’	1
What is in the Book	2
Applications of Recursion Theory	5
How to Use the Book	11
Notations and Conventions	13
I RECURSIVENESS AND COMPUTABILITY	17
I.1 Induction	18
Definitions by inductions	20
Proofs by induction	20
Recursiveness	21
Historical roots of Recursion Theory ★	22
Formal Arithmetic ★	23
Some primitive recursive functions and predicates	24
Codings of the plane	26
Elimination of primitive recursion	28
I.2 Systems of Equations	31
The formalism of equations	31
Definability by systems of equations	33
Derivability from systems of equations	36
A logical programming language ★	38
I.3 Arithmetical Formal Systems	39
Notions of representability	39
Formal systems representing the recursive functions	42
Invariant definability ★	44

	Definability of functions \star	45
I.4	Turing Machines	46
	Variations of the Turing machine model	49
	Physical Turing machines \star	51
	Finite automata \star	52
	Turing machine computability	53
	Machine-dependent programming languages \star	59
I.5	Flowcharts	61
	Unstructured programming languages \star	63
	Unlimited register, random access machines \star	64
	Flowchart computability	65
	Structured programming languages \star	68
	Programs for primitive recursion \star	70
	Petri nets \star	74
I.6	Functions as Rules	75
	λ -calculus	76
	Other formulations of the λ -calculus \star	82
	λ -definability	83
	Functional programming languages \star	86
I.7	Arithmetization	87
	Historical remarks \star	87
	Numerical tools for arithmetization	88
	The Normal Form Theorem	90
	Equivalence of the various approaches to recursiveness	97
	The basic result of the foundations of Recursion Theory	100
I.8	Church's Thesis \star	101
	Introduction to Church's Thesis	102
	Historical remarks	105
	Computers and physics	106
	Classical mechanics	107
	Probabilistic physics	109
	Computers and thought	113
	The brain	115
	Constructivism	118
	Conclusion	122
II	BASIC RECURSION THEORY	125
II.1	Partial Recursive Functions	126
	The notion of partial function	127
	Partial recursive functions	127
	Universal Turing machines and computers \star	132
	Recursively enumerable sets	134

	R.e. sets as foundation of Recursion Theory ★	143
	A programming language based on r.e. sets ★	144
II.2	Diagonalization	145
	The essence of diagonalization	145
	Recursive undecidability results	146
	Limitations of mechanisms ★	149
	Fixed-Point Theorem	152
	Limitations of formalism ★	158
	Self-reference ★	165
	Self-reproduction and cellular automata ★	170
II.3	Partial Recursive Functionals	174
	Oracle computations and Turing degrees	175
	The notion of functional	177
	Partial recursive functionals	178
	First Recursion Theorem	181
	Recursive programs ★	185
	Topological digression	186
	Iteration and fixed-points ★	192
	Models of λ -calculus (part I) ★	194
	Different notions of recursive functionals ★	196
	Higher Types Recursion Theory ★	199
	Computability on abstract structures ★	202
II.4	Effective Operations	205
	Effective operations on partial recursive functions	205
	Effective operations on total recursive functions	208
	Effective operations in general ★	210
	Recursive analysis ★	213
II.5	Indices and Enumerations ★	214
	Acceptable systems of indices	215
	Axiomatic Recursion Theory ★	221
	Models of λ -calculus (part II) ★	223
	Indices for recursive and finite sets	225
	Enumerations of classes of r.e. sets	228
	The Theory of Enumerations ★	236
II.6	Retraceable and Regressive Sets ★	238
	Retraceable versus recursive	239
	Regressive versus r.e.	242
	Existence theorems and nondeficiency sets	245
	Regressive versus retraceable	248

IIIPOST'S PROBLEM AND STRONG REDUCIBILITIES	251
III.1 Post's Problem	252
Origins of Post's Problem \star	253
Turing reducibility on r.e. sets	254
III.2 Simple Sets and Many-One Degrees	256
Many-one degrees	257
Simple sets	259
Effectively simple sets \star	263
III.3 Hypersimple Sets and Truth-Table Degrees	267
Truth-table degrees	269
Hypersimple sets	272
The permitting method \star	277
III.4 Hyperhypersimple Sets and Q-Degrees	280
Q-reducibility	281
Hyperhypersimple sets	282
Maximal sets \star	288
III.5 A Solution to Post's Problem	294
Semirecursive sets	294
η -hyperhypersimple sets	299
III.6 Creative Sets and Completeness	304
Effectively nonrecursive sets	304
Creative sets	306
Quasicreative sets \star	311
Subcreative sets \star	314
Effectively inseparable pairs of r.e. sets	316
III.7 Recursive Isomorphism Types	319
Mezoic sets and 1-degrees	320
Recursive isomorphism types	324
Recursive equivalence types and isols \star	328
III.8 Variations of Truth-Table Reducibility \star	330
Bounded truth-table degrees	331
Weak truth-table degrees	337
Other notions of reducibility \star	339
III.9 The World of Complete Sets \star	341
Relationships among completeness notions	341
Structural properties and completeness	348
III.10 Formal Systems and R.E. Sets \star	349
Formal systems and r.e. sets \star	350
Undecidability	352
Essential undecidability	353
Independent axiomatizability	357

IV HIERARCHIES AND WEAK REDUCIBILITIES	361
IV.1 The Arithmetical Hierarchy	363
The definition of truth \star	363
Truth in First-Order Arithmetic	363
The Arithmetical Hierarchy	365
The levels of the Arithmetical Hierarchy	367
Δ_2^0 sets	373
Relativizations \star	374
IV.2 The Analytical Hierarchy	375
Truth in Second-Order Arithmetic	376
The Analytical Hierarchy	377
The levels of the Analytical Hierarchy	380
Π_1^1 sets	381
Δ_1^1 sets	387
Descriptive Set Theory \star	392
Relativizations \star	394
Post's Theorem in the Analytical Hierarchy \star	395
IV.3 The Set-Theoretical Hierarchy	397
Truth in Set Theory	397
Standard structures	401
The Set-Theoretical Hierarchy	405
Δ_1^{GKP} functions	406
The levels of the Set-Theoretical Hierarchy	411
\mathcal{HF} and the Arithmetical Hierarchy	414
Absoluteness and the Analytical Hierarchy	418
Admissible sets \star	421
IV.4 The Constructible Hierarchy	422
The Constructible Hierarchy	422
The levels of the Constructible Hierarchy	424
The structure of L	425
Constructible sets of natural numbers	432
Σ_2^1 sets	437
\mathcal{HC} and the Analytical Hierarchy	441
Recursion Theory on the ordinals \star	443
Relativizations \star	444
V TURING DEGREES	447
V.1 The Language of Degree Theory	448
The join operator	448
The jump operator	450
First properties of degrees	451
The Axiom of Determinacy \star	453

V.2 The Finite Extension Method	456
Incomparable degrees	457
Embeddability results	459
The splitting method	463
Forcing the jump	467
V.3 Baire Category *	471
Topologies on total functions	472
Comeager sets	473
Baire Category and Degree Theory	477
Meager sets of degrees	481
Measure Theory and Degree Theory	484
V.4 The Coinfinite Extension Method	484
Exact pairs and ideals	485
Greatest lower bounds and least upper bounds	488
Extensions of embeddings	490
V.5 The Tree Method	493
Hyperimmune-free degrees	495
Minimal degrees	498
Minimal upper bounds *	502
König's Lemma and Π_1^0 classes *	505
Complete extensions of Peano Arithmetic *	510
V.6 Initial Segments *	516
Uniform trees	516
Minimal degrees by recursive coinfinite extensions	520
The three-element chain	523
The initial segments of the degrees *	528
V.7 Global Properties	530
Definability from parameters	530
The complexity of the theory of degrees	536
Absolute definability	540
Homogeneity	543
Automorphisms	546
V.8 Degree Theory with Jump *	550
VIMANY-ONE AND OTHER DEGREES	555
VI.1 Distributivity	555
Distributive uppersemilattices	556
Ideals of distributive uppersemilattices	558
VI.2 Countable Initial Segments	561
Finite initial segments	562
Countable initial segments	566
VI.3 Uncountable Initial Segments	569

Strong minimal covers	569
Uncountable linear orderings	570
Uncountable initial segments	571
VI.4 Global Properties	574
Characterization of the structure of many-one degrees	575
Definability, homogeneity, and automorphisms	575
The complexity of the theory of many-one degrees	577
VI.5 Comparison of Degree Theories *	582
1-degrees	582
Truth-table degrees and weak truth-table degrees	584
Elementary inequivalences	590
VI.6 Structure Inside Degrees *	591
Cylinders	591
Inside many-one degrees	594
Inside truth-table degrees	598
Inside Turing degrees	600
Bibliography	603
Notation Index	643
Subject Index	649