

Table of Contents.

Preface	v
Chapter 0: Introduction	1
Chapter 1: Preparations and basic material.....	18
1.1 Holomorphic structures and integrable connections.....	18
1.2 Gauduchon metrics.....	29
1.3 Degree maps	33
1.4 Stability of vector bundles	43
Chapter 2: Hermitian-Einstein connections and metrics	46
2.1 Definitions and first results	46
2.2 Vanishing Theorem and Chern class inequality.....	50
2.3 Stability of Hermitian-Einstein bundles	54
Chapter 3: Existence of Hermitian-Einstein metrics in stable bundles.	61
3.1 The strategy of the proof	62
3.2 The continuity method: first step	63
3.3 The continuity method: second step	71
3.4 The construction of a destabilising subsheaf.....	81
Chapter 4: The Kobayashi-Hitchin correspondence	91
4.1 Summary	91
4.2 Moduli spaces of connections	93
4.3 Moduli spaces of holomorphic structures.....	108
4.4 Isomorphy of moduli spaces	113
4.5 Local models	119
4.6 Instantons and Hermitian-Einstein connections.....	130
Chapter 5: Applications	151
5.1 Openness of the stability property	151
5.2 Dependence on the base metric	156
5.3 The natural Hermitian metric in the moduli space	167
5.4 A proof of Bogomolov's Theorem on surfaces of type VII_0	179

Chapter 6: Examples of moduli spaces	190
6.1 The algebraic case	190
6.2 Non-Kähler principal elliptic fibre bundles over curves	195
6.4 $SL(2, \mathbb{C})$ -bundles on principal elliptic bundles over curves of genus ≥ 1	200
6.5 $SL(2, \mathbb{C})$ -bundles on primary elliptic Hopf surfaces	211
Chapter 7: Appendices	217
7.1 Hermitian geometry	217
7.2 Elliptic operators	223
7.3 Sobolev spaces	229
7.4 Local diagonalisation	234
7.5 Analytic subspaces of a Banach manifold	239
Bibliography	242
Notations	251
Index	253