Contents

SERIES EDITOR'S PREFACE	xi
PREFACE	xiii
NOTATION AND ABBREVIATIONS	xvii
ORGANIZATION OF THE BOOK	xix
CHAPTER I. THE EXISTENCE OF A TRIANGLE	1
 Introduction The Fundamental Inequality History Proofs A Geometric Interpretation The Existence of a Triangle with Given Elements Some Other Results 	1 1 5 9 11 18
CHAPTER II. DUALITY BETWEEN GEOMETRIC INEQUALITIES AND INEQUALITIES FOR POSITIVE NUMBERS	26
 Introduction Geometry of the Duality (a, b, c,) ⇔ (x, y, z,) Transformations Examples Some Important Non-Negative Quadratic Forms 	26 26 27 30 33
CHAPTER III. HOMOGENEOUS SYMMETRIC POLYNOMIAL GEOMETRIC INEQUALITIES	37
 Introduction General Results of P. J. van Albada and K. B. Stolarsky Special Inequalities Best Possible Inequalities Generalization of Gerretsen's Inequalities 	37 37 39 42 45
CHAPTER IV. DUALITY BETWEEN DIFFERENT TRIANGLE INEQUALITIES AND TRIANGLE INEQUALITIES WITH (R, r, s)	49
 Some General Considerations and Some Applications Some Equivalent Forms 	49 52

vi CONTENTS

	ER V. TRANSFORMATIONS FOR THE ANGLES OF A IANGLE	64
		01
1.	Some Applications of Pexider's Functional Equation in	- 4
_	Geometry	64
2.	Some Applications	70
3.	On the Obtaining of Analytic Inequalities from Geometric Ones	71
СНАРТ	ER VI. SOME TRIGONOMETRIC INEQUALITIES	74
0.	Introduction	74
1.	Asymmetric Trigonometric Inequalities	74
2.	Some Trigonometric Identities	85
3.	Some Applications	90
4.	Open Questions	98
СНАРТ	ER VII. SOME OTHER TRANSFORMATIONS	101
0.	Introduction	101
1.	Square-Root Transformation	102
2.	Generalizations of the Finsler-Hadwiger Inequality and	102
2.	Applications	104
3.	Some Trigonometric Transformations	106
4.	The Median-Dual Transformation and Its Generalizations	109
5.	Transformations T_a , T_a and T_{a-1}	112
6.	Parallelogram Transformations	121
7.	A Transformation for Acute Triangles	117
СНАРТ	ER VIII. CONVEX FUNCTIONS AND GEOMETRIC	
	EQUALITIES	121
1. 2.	Jensen's and Related Inequalities and Geometric Inequalities Majorization and Geometric Inequalities and Geometric	121
۷.	Inequalities	121
	2.1. Inequalities for the Sides of a Triangle and Polygon	129
	2.2. Inequalities for the Angles of a Triangle and Polygon	133
	2.2. Inequalities for the Angles of a Triangle and Polygon 2.3. Majorization and Other Elements of a Triangle	138
		139
0	2.4. Majorization and Isoperimetric-Type Inequalities	140
3.	Applications of Some Other Inequalities for Convex Functions	140
СНАРТ	ER IX. MISCELLANEOUS INEQUALITIES WITH ELE-	
	ENTS OF A TRIANGLE	144
1.	Inequalities Involving only the Sides of a Triangle	144

CONTENTS	vii

	2.	Inequalities for the Angles of a Triangle	149
		2.1. Bager's Graphs	149
		2.2. Miscellaneous Inequalities for the Angles of a Triangle	154
	3.	Inequalities with (R, r, s)	165
	4.	Inequalities for the Sides and the Angles of a Triangle	168
	5.	Inequalities with (a, b, c) and $(R, r, s \text{ or } F)$	170
	6.	Inequalities Involving A , B , C and R , r , s or F	180
	7.	Inequalities with (a, b, c) , (A, B, C) and $(R, r, s \text{ or } F)$	190
	8.	Inequalities for the Radii of Excircles and Other Elements of a	
		Triangle	192
	9.	Inequalities Involving Altitudes and Other Elements of a	
		Triangle	200
		Inequalities with Medians and Other Elements of a Triangle	209
	11.	Inequalities with Angle Bisectors and Other Elements of a	
		Triangle	217
	12.	Inequalities with Some Elements of a Triangle Extended to the	
		Circumcircle	229
CHA	PTE	ER X. SPECIAL TRIANGLES	232
	1.	On the Triangles Satisfying $a^2 + b^2 + c^2 \ge kR^2$	232
	2.	On Some Inequalities for Acute (Non-Obtuse) Triangles	240
		2.0. Introduction	240
		2.1. A Question of Ono	240
		2.2. An Inequality of Gridasow	242
		2.3. An Inequality of Walker	247
	3.	On Some Inequalities for Obtuse (Non-acute) Triangles	251
		3.1. Emmerich's Inequality	251
		3.2. Inequalities of Bottema and Groenman	253
	4.	Some Other Classes of Special Triangles	255
		4.0. Introduction	255
		4.1. Inequalities for Triangles of Bager's Type I or II	256
		4.2. Inequalities of S. G. Guba for Special Triangles	261
		4.3. Inequalities of I. Paasche for Special Triangles	263
		4.4. Some Further Remarks	265
	5.	Some Other Results for Special Triangles	267
СНА	PTE	R XI. TRIANGLE AND POINT	278
	1.	Some Applications of a Generalization of the Leibniz Identity	278
	2.	Some Transformations	293
		2.1. Inversion	293
		2.2. Reciprocation	295
		2.3. Isogonal Conjugates	295

viii CONTENTS

* * * * * * * * * * * * * * * * * * *	
2.4. Pedal Triangle	296
3. Some Further Remarks on the Polar Moment of Inertia In-	206
equality	296
4. Two Simple Methods for Generating Inequalities Involving	301
Triangle and Point	
4.1. Applications of Jensen's Inequality for Convex Function	301
and of the Inequality for Means 4.2. Triangle Inequalities from the Triangle Inequality	309
and the second s	313
5. Erdös-Mordell's Related Inequalities6. Miscellaneous Inequalities Involving Characteristic Points	319
7. Miscellaneous Inequalities Involving Arbitrary Points	333
7. Wilsochuncous modulation myorang careers,	
CHAPTER XII. INEQUALITIES WITH SEVERAL TRIANGLES	340
1. Inequalities Related to Two Triangles Inscribed One in the	
Other	340
2. Some Other Inequalities with Triangles Connected to the Given	en 247
Triangle	347
3. The Neuberg-Pedoe and the Oppenheim Inequalities	354 354
3.0. Introduction	354
3.1. The Neuberg-Pedoe and the Oppenheim Inequality	333
3.2. Comment by O. Bottema: On the Mixed Area of Two	357
Triangles 3.3. On a Result of Carlitz	358
3.4. Sharpening the Neuberg-Pedoe and the Oppenheim	550
Inequality	359
3.5. Further Generalizations of the Neuberg-Pedoe Inequali	ity 364
3.6. Further Generalizations of the Oppenheim Inequality	366
4. On O. Bottema's Inequality for Two Triangles	371
5. Miscellaneous Inequalities Involving Elements of Several	
Triangles	375
CHAPTER XIII. THE MÖBIUS-NEUBERG AND THE MÖBIUS-	385
POMPEIU THEOREMS	383
CHAPTER XIV. INEQUALITIES FOR QUADRILATERALS	401
CHAPTER XIV. INEQUALITIES FOR QUADRIENTER LES	
CHAPTER XV. INEQUALITIES FOR POLYGONS	412
THE DESCRIPTION A CIDCLE	433
CHAPTER XVI. INEQUALITIES FOR A CIRCLE	433
CHAPTER XVII. PARTICULAR INEQUALITIES IN PLANE	
CEOMETRY	443

CONTENTS		12

1. S	ome Isoperimetric Inequalities	443
2. V	arious Particular Inequalities	448
CHAPTER	XVIII. INEQUALITIES FOR SIMPLEXES IN E^n $(n \ge 2)$	463
1. Ir	nequalities for r , ρ_i , $h_i F_i$ $(i = 1, 2,, n + 1)$ in a Simplex	463
	nequalities for the Simplex and a Point	473
	Other Inequalities for a Simplex	515
	nequalities for Two (or More) Simplexes	533
CHAPTER	XXIX. INEQUALITIES FOR TETRAHEDRA	545
1. I	nequalities for the Edge Lengths and Face Areas of a	
Γ	Tetrahedron	545
2. I	nequalities for the Volume, the Circumradius and Other	
E	Elements of a Tetrahedron	553
3. (Other Inequalities for Tetrahedra	560
4. I	nequalities for Special Tetrahedra	567
CHAPTER	RXX. OTHER INEQUALITIES IN E^n $(n \ge 2)$	576
1. I	nequalities for Convex Polyhedra	576
2. I	nequalities for Prisms	592
3. I	nequalities for Pyramids	602
4. I	nequalities for the Regular Convex Polyhedra and Polyhedra	
I	somorphic to Them	620
5. I	nequalities for Quadric Surfaces	626
6. I	nequalities for Spherical Triangles	636
	Other Inequalities in E^3	647
	nequalities for Convex Polytopes in $(E^n (n \ge 2))$	653
9. (Other Inequalities in E^n $(n \ge 2)$	659
APPENDI	x	677
1. (Comments, Additions and Corrections for GI	677
2. S	Supplement to Chapters I–XX	682
NAME IN	DEX	695
SUBJECT	INDEX	706