CONTENTS

Preface ix

Chapter I. Introduction 1

1.1 Vector Bundles 1
1.1.1 Vector Bundles 1
1.1.2 Connections 5
1.2 Harmonic Maps 9
1.2.1 Energy 10
1.2.2 Tension Field 11
1.2.3 The First Variational Formula 13
1.2.4 Examples of Harmonic Maps 15
1.3 A Bochner Type Formula 17
1.3.1 Hodge-Laplace Operator and Weitzenböck Formula 17
1.3.2 A Bochner Type Formula and Its Applications 22
1.4 Basic Properties of Harmonic Maps 28
1.4.1 Maximum Principle 28
1.4.2 Unique Continuation Theorems 32
1.4.3 Second Variational Formula and Stable Harmonic Maps 33

Chapter II. Conservation Law 39

2.1 Stress-Energy Tensor and Conservation Law 39
2.2 Monotonicity Formula 43
2.3 Applications of Conservation Law to Liouville type Theorems 46
2.4 Further Generalizations 49

Chapter III. Harmonic Maps and Gauss Maps 61

3.1 Generalized Gauss Maps 61
3.2 Cone-like Harmonic Maps 64
3.3 Generalized Maximum Principle 70
3.4 Estimates of Image Diameter and its Applications 76

3.5 Gauss Image of a Space-Like Hypersurface in Minkowski Space 79
3.6 Gauss Image of a Space-Like Submanifold in Pseudo-Euclidean Space 86
3.6.1 Geometry of $\Re_{IV}(2)$ 87
3.6.2 Gauss Map 89
3.6.3 Gauss Image of a Space-like Surface in $\mathbb{R}^4_2$ 91

Chapter IV. Harmonic Maps and Holomorphic Maps **97**

4.1 Partial Energies 97
4.2 Harmonicity of Holomorphic Maps 99
4.3 Holomorphicity of Harmonic Maps 102

Chapter V. Existence, Nonexistence and Regularity **121**

5.1 Direct Method of the Calculus of Variations 121
5.2 Regularity Theorems 124
5.3 Nonexistence and Existence 127
5.4. Regularity Results of Harmonic Maps into Positively Curved Manifolds 132

Chapter VI. Equivariant Harmonic Maps **147**

6.1 Riemannian Submersions and Equivariant Harmonic Maps 147
6.2 Reduction Theorems 152
6.3 Equivariant Variational Formulas 156
6.4 On Harmonic Representatives of $\Pi_m(S^m)$ 162
6.4.1 ODE of Smith's Construction 163
6.4.2 The Solvability of ODE (6.31) and (6.33) 166
6.4.3 Application of Smith's Construction 179
6.4.4 Another Construction of Equivariant Maps 181
6.4.5 The Solvability of ODE (6.63) and (6.65) 186
6.4.6 On Harmonic Representatives of Homotopy Group of the Higher Dimensional Sphere 191
6.5 Harmonic Maps via Isoparametric Maps 193
6.6 Harmonic Maps of Projective Spaces 202
6.6.1 Harmonic Maps from $\mathbb{QP}n-1$ into S^{m1} 205
6.6.2 Harmonic Maps from $\mathbb{QP}n-1$ into $\mathbb{QP}^{m-1}$ 206

6.7 Equivariant Boundary Value Problems 209
6.7.1 The Reduced PDE 209
6.7.2 The Solvability of PDE 212
6.7.3 Construction of Equivariant Maps into $\mathbb{CP}^2$ 216
6.7.4 Heat Flow 217
6.7.5 Global Existence and Subconvergence 223

References **227**

Index **239**