## Contents

Introduction	
Chapter 1 First results on rigid local systems	
1.0 Generalities concerning rigid local systems over C	13
1.1 The case of genus zero	14
1.2 The case of higher genus	18
1.3 The case of genus one	23
1.4 The case of genus one: detailed analysis	24
CHAPTER 2 The theory of middle convolution	
2.0 Transition from irreducible local systems on open sets of	
$\mathbb{P}^1$ to irreducible middle extension sheaves on $\mathbb{A}^1$ .	35
2.1 Transition from irreducible middle extension sheaves on	
$\mathbb{A}^1$ to irreducible perverse sheaves on $\mathbb{A}^1$	36
2.2 Review of $\mathrm{D}^{\mathrm{b}}_{\mathrm{c}}(\mathrm{X},\overline{\mathbb{Q}}_{\ell})$	38
2.3 Review of perverse sheaves	39
2.4 Review of Fourier Transform	45
2.5 Review of convolution	45
2.6 Convolution operators on the category of perverse sheaves: middle convolution	47
2.7 Interlude: middle direct images (relative dimension one)	56
2.8 Middle additive convolution via middle direct image	57
2.9 Middle additive convolution with Kummer sheaves 2.10 Interpretation of middle additive convolution via Fourier	59
Transform	65
2.11 Invertible objects on A <sup>1</sup> in characteristic zero	72
2.12 Musings on $\star_{ m mid}$ -invertible objects in ${ m I\!P}$ in the ${ m G}_{ m m}$ case	75
2.13 Interlude: surprising relations between $*_{ ext{mid}}$ on $\mathbb{A}^1$ and	
on $\mathfrak{G}_{m}$	81
2.14 Interpretive remark: Fourier-Bessel Transform	83
2.15 Questions about the situation in several variables	84
2.16 Questions about the situation on elliptic curves	84
2.17 Appendix 1: the basic lemma on end-exact functors	87
2.18 Appendix 2: twisting representations by characters	- 88

CHAPTER 3 Fourier Transform and rigidity	
3.0 Fourier Transform and index of rigidity	91
3.1 Lemmas on representations of inertia groups	94
3.2 Interlude: the operation ⊗ <sub>mid</sub>	99
3.3 Applications to middle additive convolution	100
3.4 Some open questions about local Fourier Transform	106
Chapter 4 Middle convolution: dependence on parameters	
4.0 Good schemes	111
4.1 The basic setting	111
4.2 Basic results in the basic setting	112
4.3 Middle convolution in the basic setting	116
Chapter 5 Structure of rigid local systems	
5.0 Cohomological rigidity	121
5.1 The category T $_{\ell}$ , and the functors MC $_{\chi}$ and MT $_{\Sigma}$	121
5.2 The main theorem on the structure of rigid local systems	125
5.3 Applications and Interpretations of the main theorem	131
5.4 Some open questions	131
5.5 Existence of universal families of rigids with given local	
monodromy	132
5.6 Remark on braid groups	143
5.7 Universal families without quasiunipotence	143
5.8 The complex analytic situation	144
5.9 Return to the original question	146
Chapter 6 Existence algorithms for rigids	
6.0 Numerical invariants	153
6.1 Numerical incarnation: the group NumData	156
6.2 A compatibility theorem	160
6.3 Realizable and plausible elements	161
6.4 Existence algorithm for rigids	164
6.5 An example	165
6.6 Open questions	166
6.7 Action of automorphisms	166
6.8 A remark and a question	167

CHAPTER 7 Diophantine aspects of rigidity	
7.0 Diophantine criterion for irreducibility	169
7.1 Diophantine criterion for rigidity	170
7.2 Appendix: a counterexample	174
CHAPTER 8 Motivic description of rigids	
8.0 The basic setting	183
8.1 Interlude: Kummer sheaves	184
8.2 Naive convolution on ( $\mathbb{A}^{1}$ - {T <sub>1</sub> ,, T <sub>n</sub> }) <sub>SN,n,\ell</sub> .	185
8.3 Middle convolution on ( $\mathbb{A}^1$ - {T <sub>1</sub> ,, T <sub>n</sub> }) $\mathbb{S}_{N,n,\ell}$ .	188
8.4 "Geometric" description of all tame rigids with	
quasi-unipotent local monodromy	194
8.5 A remark and a question	196
CHAPTER 9 Grothendieck's p-curvature conjecture for rigids	
9.0 Introduction	197
9.1 Review of Grothendieck's p-curvature conjecture	197
9.2 Interlude: Picard-Fuchs equations and some variants	199
9.3 The main result of [Ka-ASDE] and a generalization	202
9.4 Application to rigid local systems	211
9.5 Comments and questions	216
References	219