Contents

	List	or Symbols	х
	\mathbf{Pre}	face	x
	Int	roduction	xvi
1	Bac	ekground from Algebraic Number Theory.	1
	1.1	Quadratic Fields: Integers and Units	
	1.2	The Arithmetic of Ideals in Quadratic Fields	. 9
	1.3	The Class Group and Class Number	. 14
	1.4	Reduced Ideals	. 19
	1.5	Quadratic Orders	
	1.6	Powerful Numbers: An Application of Real Quadratics	. 30
2	Cor	ntinued Fractions Applied to Quadratic Fields.	41
	2.1	Continued Fractions and Real Quadratics: The Infrastructure	
	2.2	The Continued Fraction Analogue for Complex Quadratics	. 63
3	Dio	phantine Equations and Class Numbers.	67
	3.1	Class Numbers and Complex Quadratics	
	3.2	Real Quadratics and Diophantine Equations	
	3.3	Reduced Ideals and Diophantine Equations	
	3.4	Class Numbers and Real Quadratics	
	3.5	Halfway to a Solution	. 90
4	Pri	me-Producing Polynomials.	105
	4.1	Complex Prime-Producers	. 108
	4.2	Real Prime-Producers	
	4.3	Density of Primes	. 14
5	Class Numbers: Criteria and Bounds.		
	5.1	Factoring Rabinowitsch	. 149
	5.2	Class Number One Criteria	. 158
	5.3	Class Number Bounds via the Divisor Function	. 163
	5.4	The GRH: Relevance of the Riemann Hypothesis	. 172
6	Am	biguous Ideals.	187
	6.1	Ambiguous Cycles in Real Orders: The Palindromic Index	. 187
	69	Evnoport Two	. 199

7	Influence of the Infrastructure. 7.1 Quadratic Residue Covers	
8	Algorithms. 8.1 Computation of the Class Number of a Real Quadratic Field 8.2 Cryptology	257
\mathbf{A}	PPENDIX A: Tables.	271
	Table A1: This is a list of all positive fundamental radicands with class number $h_{\Delta} = 1$ and period length ℓ , of the simple continued fraction expansion of the principal class, less than 24, known to be unconditionally complete whereas Table A1 is complete with one GRH-ruled out exception, as are Tables A2-A4, A6-A7 and A9	271
	Table A2: This is a subset of Table A1 with $D \equiv 1 \pmod{8} \dots \dots$	
	Table A3: $h_{\Delta} = 2$ for fundamental radicands $D > 0$ with $\ell \le 24$	
	Table A4: This is a list of all fundamental radicands of ERD-type with class groups of exponent 2, broken down into three parts depending on congruence modulo 4 of the radicand	278
	 Table A5: This three-part table is an illustration of a computer run done for the proof of Theorem 6.2.2	
	having no split primes less than the Minkowski bound	
	Table A8: This is a list of all fundamental discriminants $\Delta \equiv 1 \pmod 8$ of ERD-type with class number less than 24, and is known to be un-	
	conditionally complete	285
	with class number 2	286
A]	PPENDIX B: Fundamental Units of Real Quadratic Fields. This list is broken up into three parts according to congruence modulo 4 of fundamental radicands less than 2 · 10 ³	287
A]	PPENDIX C: Class Numbers of Real Quadratic Fields. This table is presented in matrix form with each entry describing a specified class number together with the norm of the fundamental unit with radicands less than 10 ⁴	313
A l	PPENDIX D: Class Numbers of Complex Quadratic Fields (and their class group structure).	d
	This is a table of fundamental radicands $D < 0$ for $ D < 2 \cdot 10^3$, together with h_{Δ} and the class group structure as a product of cyclic groups	335

Contents

APPENDIX E: A Gazetteer of Forms We give an overview of the classical theory of binary quadratic forms and the interrelationships with ideal theory	347
APPENDIX F: Analytic Considerations This provides background facts from analytic number theory. In particular this gives a brief overview of the required material for Chapter Five, section four	355
BIBLIOGRAPHY.	359
INDEX.	383