Contents

Preface v

Basic Notation vii

1 Borsuk's Problem 1
§1 Introduction 1
§2 The Perkal-Eggleston Theorem 2
§3 Some Remarks 7
§4 Larman's Problem 8
§5 The Kahn-Kalai Phenomenon 9

2 Finite Packing Problems 15
§1 Introduction 15
§2 Supporting Functions, Area Functions, Minkowski Sums, Mixed Volumes, and Quermassintegrals 16
§3 The Optimal Finite Packings Regarding Quermassintegrals 18
§4 The L. Fejes Tóth-Betke-Henk-Wills Phenomenon 23
§5 Some Historical Remarks 34

3 The Venkov-McMullen Theorem and Stein's Phenomenon 37
§1 Introduction 37
§2 Convex Bodies and Their Area Functions 37
§3 The Venkov-McMullen Theorem 43

§4 Stein's Phenomenon 50
§5 Some Remarks 53

4 Local Packing Phenomena **55**
§1 Introduction 55
§2 A Phenomenon Concerning Blocking Numbers and Kissing Numbers 57
§3 A Basic Approximation Result 62
§4 Minkowski's Criteria for Packing Lattices and the Densest Packing Lattices 63
§5 A Phenomenon Concerning Kissing Numbers and Packing Densities 70
§6 Remarks and Open Problems 81

5 Category Phenomena **83**
§1 Introduction 83
§2 Gruber's Phenomenon 84
§3 The Aleksandrov-Busemann-Feller Theorem 85
§4 A Theorem of Zamfirescu 90
§5 The Schneider-Zamfirescu Phenomenon 91
§6 Some Remarks 98

6 The Busemann-Petty Problem **99**
§1 Introduction 99
§2 Steiner Symmetrization 100
§3 A Theorem of Busemann 101
§4 The Larman-Rogers Phenomenon 108
§5 Schneider's Phenomenon 113
§6 Some Historical Remarks 121

7 Dvoretzky's Theorem **123**
§1 Introduction 123
§2 Preliminaries 123
§3 Technical Introduction 126
§4 A Lemma of Dvoretzky and Rogers 128
§5 An Estimate for $\sigma_V(A_V)$ 130
§6 β-nets and ϵ-spheres 134
§7 A Proof of Dvoretzky's Theorem 136
§8 An Upper Bound for $M(n, \epsilon)$ 137
§9 Some Historical Remarks 141

Bibliography **143**

Index **155**