

Contents

1 Preliminaries	1
1.1 Introduction	1
1.2 What is a vector bundle?	4
1.3 What is a connection?	8
1.4 The curvature of a connection	15
1.5 Characteristic classes	18
1.6 The universal bundle	22
1.7 Classification of connections	28
1.8 Hodge theory	33
2 Spin geometry on four-manifolds	39
2.1 Euclidean geometry and the spin groups	39
2.2 What is a spin structure?	43
2.3 Almost complex and spin^c structures	45
2.4 Clifford algebras	47
2.5 The spin connection	50
2.6 The Dirac operator	55
2.7 The Atiyah-Singer Index Theorem	58
3 Global analysis of the Seiberg-Witten equations	65
3.1 The Seiberg-Witten equations	65
3.2 The moduli space	67
3.3 Compactness of the moduli space	71
3.4 Transversality	73
3.5 Topology of four-manifolds	82
3.6 Seiberg-Witten invariants	87
3.7 Dirac operators on Kähler surfaces	89
3.8 Invariants of Kähler surfaces	95
Bibliography	101
Index	104