

Contents

Introduction	vii
1 The Laplacian on a Riemannian Manifold	1
1.1 Basic Examples	2
1.1.1 The Laplacian on S^1 and \mathbf{R}	3
1.1.2 Heat Flow on S^1 and \mathbf{R}	5
1.2 The Laplacian on a Riemannian Manifold	10
1.2.1 Riemannian Metrics	10
1.2.2 L^2 Spaces of Functions and Forms	14
1.2.3 The Laplacian on Functions	17
1.3 Hodge Theory for Functions and Forms	22
1.3.1 Analytic Preliminaries	22
1.3.2 The Heat Equation Proof of the Hodge Theorem for Functions	27
1.3.3 The Hodge Theorem for Differential Forms	33
1.3.4 Regularity Results	35
1.4 De Rham Cohomology	39
1.5 The Kernel of the Laplacian on Forms	46
2 Elements of Differential Geometry	52
2.1 Curvature	52
2.2 The Levi-Civita Connection and Bochner Formula	63
2.2.1 The Levi-Civita Connection	63
2.2.2 Weitzenböck Formulas and Gårding's Inequality	67
2.3 Geodesics	77
2.4 The Laplacian in Exponential Coordinates	83
3 The Construction of the Heat Kernel	88
3.1 Preliminary Results for the Heat Kernel	88
3.2 Construction of the Heat Kernel	90
3.2.1 Construction of the Parametrix	90
3.2.2 The Heat Kernel for Functions	94
3.3 The Asymptotics of the Heat Kernel	99

3.4	Positivity of the Heat Kernel	106
4	The Heat Equation Approach to the Atiyah-Singer Index Theorem	109
4.1	The Chern-Gauss-Bonnet Theorem	109
4.1.1	The Heat Equation Approach	110
4.1.2	Proof of the Chern-Gauss-Bonnet Theorem	114
4.2	The Hirzebruch Signature Theorem and the Atiyah-Singer Index Theorem	126
4.2.1	A Survey of Characteristic Forms	126
4.2.2	The Hirzebruch Signature Theorem	132
4.2.3	The Atiyah-Singer Index Theorem	137
5	Zeta Functions of Laplacians	142
5.1	The Zeta Function of a Laplacian	142
5.2	Isospectral Manifolds	149
5.3	Reidemeister Torsion and Analytic Torsion	151
5.3.1	Reidemeister Torsion	151
5.3.2	Analytic Torsion	152
5.3.3	The Families Index Theorem and Analytic Torsion	161
	Bibliography	165
	Index	170