

Table of Contents

Foreword	V
Preface	VII
1. Algebraic Preliminaries	1
1.1 Groups, Rings and Fields	1
1.2 Euclidean Division and Pseudo-Division	8
1.3 The Euclidean Algorithm	10
1.4 Resultants and Subresultants	18
1.5 Polynomial Remainder Sequences	21
1.6 Primitive Polynomials	25
1.7 Squarefree Factorization	28
Exercises	32
2. Integration of Rational Functions	35
2.1 The Bernoulli Algorithm	36
2.2 The Hermite Reduction	39
2.3 The Horowitz–Ostrogradsky Algorithm	45
2.4 The Rothstein–Trager Algorithm	47
2.5 The Lazard–Rioboo–Trager Algorithm	49
2.6 The Czichowski Algorithm	53
2.7 Newton–Leibniz–Bernoulli Revisited	54
2.8 Rioboo’s Algorithm for Real Rational Functions	59
2.9 In-Field Integration	70
Exercises	72
3. Differential Fields	73
3.1 Derivations	73
3.2 Differential Extensions	77
3.3 Constants and Extensions	83
3.4 Monomial Extensions	88
3.5 The Canonical Representation	97
Exercises	102

4. The Order Function	105
4.1 Basic Properties	105
4.2 Localizations	108
4.3 The Order at Infinity	113
4.4 Residues and the Rothstein–Trager Resultant	116
Exercises	124
5. Integration of Transcendental Functions	125
5.1 Elementary and Liouvillian Extensions	125
5.2 Outline and Scope of the Integration Algorithm	130
5.3 The Hermite Reduction	134
5.4 The Polynomial Reduction	136
5.5 Liouville’s Theorem	138
5.6 The Residue Criterion	143
5.7 Integration of Reduced Functions	150
5.8 The Primitive Case	153
5.9 The Hyperexponential Case	156
5.10 The Hypertangent Case	159
5.11 The Nonlinear Case with no Specials	168
5.12 In-Field Integration	171
Exercises	174
6. The Risch Differential Equation	177
6.1 The Normal Part of the Denominator	177
6.2 The Special Part of the Denominator	182
6.3 Degree Bounds	189
6.4 The SPDE Algorithm	198
6.5 The Non-Cancellation Cases	202
6.6 The Cancellation Cases	207
Exercises	212
7. Parametric Problems	213
7.1 The Parametric Risch Differential Equation	213
7.2 The Limited Integration Problem	241
7.3 The Parametric Logarithmic Derivative Problem	246
Exercises	251
8. The Coupled Differential System	253
8.1 The Primitive Case	255
8.2 The Hyperexponential Case	257
8.3 The Nonlinear Case	258
8.4 The Hypertangent Case	260

9. Structure Theorems	265
9.1 The Module of Differentials	265
9.2 Rosenlicht's Theorem	272
9.3 The Risch Structure Theorems	278
9.4 The Rothstein–Caviness Structure Theorem	288
Exercises	292
References	293
Index	297