

Contents

1	Concepts and History	1
1.1	Basic Concepts	1
1.1.1	Prime Numbers	2
1.1.2	Amicable and Perfect Numbers	10
1.1.3	Amicable k-Tuplets	19
1.1.4	Aliquot k-Cycles	21
1.2	The History of Amicable Numbers	24
1.2.1	Ancient Times	25
1.2.2	Around Euler's Time	28
1.2.3	The Modern Computer Era	31
1.3	Brief History of Perfect and Sociable Numbers	34
2	Mathematical Tools	39
2.1	Computational Complexity Theory	39
2.2	Primality Testing	46
2.2.1	Basic Idea	46
2.2.2	Probable Primes and Strong Pseudoprimality Tests	48
2.2.3	Lucas Sequences and Lucas Tests	54
2.2.4	Primality Testing in Maple	63
2.2.5	Elliptic Curve Test	67
2.2.6	Computational Complexity of Primality Testing	71
2.3	Integer Factorization	74
2.3.1	Computational Complexity of Integer Factorization	74
2.3.2	Factoring by Trial Divisions	77
2.3.3	Continued FRACTION Method (CFRAC)	80
2.3.4	Pollard's " <i>rho</i> " and " <i>p</i> - 1" Algorithms	84
2.3.5	Lenstra's Elliptic Curve Algorithm	87
2.3.6	Progress on Factoring Fermat Numbers	89

2.3.7	Integer Factorization in Maple	92
2.4	Diophantine Equations	97
2.4.1	Continued Fraction Approach	98
2.4.2	Combinatorial Approach	100
3	Exhaustive Numerical Methods	103
3.1	Numerical Methods for Perfect Numbers	104
3.2	Computing Different Types of Perfect Numbers	107
3.3	Computing Odd Perfect Numbers	111
3.4	Numerical Methods for Amicable Numbers	113
3.5	te Riele's Seminumerical Method for Amicable Pairs	118
3.6	Computing Reduced Amicable Pairs	121
3.7	Numerical Methods for Aliquot k-Cycles	128
4	Algebraic Assumption Methods	137
4.1	Thabit's Algebraic Assumption Method	137
4.2	Euler's Version of Thabit's Method	140
4.3	Analogue of Thabit's Method	146
4.4	Some Conjectures on Amicable Pairs	151
4.5	Algebraic Rules for Aliquot k-Cycles	153
4.5.1	Algebraic Rule for Aliquot 3-Cycles	154
4.5.2	Algebraic Rules for Aliquot 4-Cycles	163
5	Algebraic Constructive Methods	169
5.1	A Special Algebraic Constructive Method	169
5.2	A General Algebraic Constructive Method	180
5.3	Two Other More General Constructive Methods	189
5.3.1	Forward Methods	190
5.3.2	Backward Methods	199
5.4	Algebraic Methods for Amicable k-Tuplets	201
5.4.1	Methods for Amicable Triplets	202
5.4.2	Methods for Amicable k-Tuplets	203
5.5	A Practical Application in Cryptography	207
6	Conclusions and Open Problems	215
6.1	Summary	215
6.1.1	The History of Amicable Numbers	215
6.1.2	Mathematical Tools for Amicable Numbers	217
6.1.3	Exhaustive Numerical and Semi-numerical Methods	219

6.1.4	Algebraic Assumption Methods	220
6.1.5	Algebraic Constructive Methods	221
6.1.6	Computer-Aided Proof	221
6.2	Some Open Problems	222
6.2.1	Problems Related to Amicable Numbers	222
6.2.2	Problems in General Computational Number Theory	225
Bibliography		233
A Basic Number Theory		245
A.1	What is Number Theory	245
A.2	Divisibility	248
A.3	Euclid's Algorithm	252
A.4	Arithmetic Functions	256
A.5	Congruences	259
B Introduction to the Maple numtheory/combinat Packages		267
B.1	Basic Concepts	267
B.2	Number-Theoretic Functions	268
B.2.1	Basic Arithmetic Functions	270
B.2.2	Modular Arithmetic Functions	272
B.2.3	Primality Testing Related Functions	273
B.2.4	Integer Factorization Related Functions	278
B.2.5	Continued Fractions	281
B.2.6	Legendre/Jacobi Symbols	287
B.3	Combinatorial Functions	289
B.4	Programming in Maple	293
C Selected Maple Programs		299
C.1	Programs for Numerical Methods	299
C.2	Programs for Algebraic Assumption Methods	307
C.3	Programs for Algebraic Constructive Methods	316
Index		335