CONTENTS

Pr	etace	XV
Ch	apter 1. Simplest Classical Variational Problems	1
§1	Equations of Extremals for Functionals	1
§ 2	Geometry of Extremals 2.1. The Zero-Dimensional and One-Dimensional Cases	5
	 2.2. Some Examples of the Simplest Multidimensional Functional. The Volume Functional 2.3. The Classical Plateau Problem in Dimension 2 2.4. The Second Fundamental Form on the Riemannian Submanifold 2.5. Local Minimality 2.6. First Examples of Globally Minimal Surfaces 	12 14 16 18
	apter 2. Multidimensional Variational Problems and Extraordinary o) Homology Theory	23
	 The Multidimensional Plateau Problem and Its Solution in the Class of Mapping on Spectra of Manifolds with Fixed Boundary 3.1. The Classical Formulations (Finding the Absolute Minimum) 3.2. The Classical Formulations (Finding a Relative Minimum) 3.3. Difficulties Arising in the Minimization of the Volume Functional volk for k > 2. Appearance on Nonremovable Strata of Small Dimensions 3.4. Formulations of the Plateau Problem in the Language of the Usual Spectral Homology 3.5. The Classical Multidimensional Plateau Problem (the Absolute Minimum) and the Language of Bordism Theory 3.6. Spectral Bordism Theory as an Extraordinary Homology Theory 3.7. The Formulation of the Solution to the Plateau Problem (Existence of the Absolute Minimum in Spectral Bordism Classes) 	23 25 26 29 30 35
§4	Extraordinary (Co)Homology Theories Determined for "Surfaces with Singularities" 4.1. The Characteristic Properties of (Co)Homology Theories	4:

viii Contents

	4.2. Extraordinary (Co)Homology Theories	
	for Finite Cell Complexes	43
	4.3. The Construction of Extraordinary (Co)Homology Theories for "Surfaces with Singularities" (on Compact Sets)	45
	4.4. Verifying the Characteristic Properties of the Constructed	710
	Theories	46
	4.5. Additional Properties of Extraordinary Spectral Theories	48
	4.6. Reduced (Co)Homology Groups on "Surfaces with Singularities"	49
§5	The Coboundary and Boundary of a Pair of Spaces (X, A)	49
	5.1. The Coboundary of a Pair (X, A)	5(
	5.2. The Boundary of a Pair (X, A)	5(
§6	Determination of Classes of Admissible Variations of Surfaces in	
	Terms of (Co)Boundary of the $Pair(X, A)$	5 1
	6.1. Variational Classes $h(A, L, L')$ and $h(A, L)$	51
	6.2. The Stability of Variational Classes	53
§ 7	Solution of the Plateau Problem (Finding Globally Minimal	
	Surfaces (Absolute Minimum) in the Variational Classes	_
	h(A, L, L') and $h(A, L)$	54
	7.1. The Formulation of the Problem	54
	7.2. The Basic Existence Theorem for Globally Minimal Surfaces. Solution of the Plateau Problem	56
	7.3. A Rough Outline of the Existence Theorem	59
02	Solution of the Problem of Finding Globally Minimal Surfaces	•
yo	in Each Homotopy Class of Multivarifolds	61
~1	••	
	napter 3. Explicit Calculation of Least Volumes (Absolute	62
	inimum) of Topologically Nontrivial Minimal Surfaces	
§ 9	Exhaustion Functions and Minimal Surfaces	62
	9.1. Certain Classical Problems 9.2. Bordisms and Exhaustion Functions	62 64
	9.2. Bordisms and Exhaustion runctions 9.3. GM-Surfaces	65
	9.4. Formulation of the Problem of a Lower Estimate of the	0,
	Minimal Surface Volume Function	66
810	Definition and Simplest Properties of the Deformation Coefficient	
3-	of a Vector Field	6
81	1 Formulation of the Basic Theorem for the Lower Estimate of the	
31.	Minimal Surface Volume Function	68
	11.1. Functions of the Interaction of a Globally Minimal Surface	•
	with a Wavefront	69
	11.2. Formulation of the Basic Volume Estimation Theorem	69
§1:	2 Proof of the Basic Volume Estimation Theorem	7
81:	3 Certain Geometric Consequences	7

Contents ix

	3.1. On the Least Volume of Globally Minimal Surfaces Passing	77
	through the Centre of a Ball in Euclidean Space	77
	3.2. On the Least Volume of Globally Minimal Surfaces Passing	79
	through a Fixed Point in a Manifold	19
	3.3. On the Least Volume of Globally Minimal Surfaces Formed by the Integral Curves of a Field v	80
		00
§14	Jullity of Riemannian, Compact, and Closed Manifolds. Geodesic	
	Sullity and Least Volumes of Globally Minimal Surfaces of	0.4
	Realizing Type	81
	4.1. The Definition of the Nullity of a Manifold	81
	4.2. The Theorem on the Relation of Nullity with the Least	0.0
	Volumes of Surfaces of Realizing Type	83
	4.3. The Proof of the Reifenberg Conjecture Regarding the	
	Existence of a Universal Upper Estimate of the "Complexity"	0.6
	on the Singular Points of Minimal Surfaces of Realizing Type	86
$\S15$	Certain Topological Corollaries. Concrete Series of Examples of	
	Globally Minimal Surfaces of Nontrivial Topological Type	88
	5.1. Globally Minimal Surfaces Realizing Nontrivial (Co)Cycles	
	in Symmetric Spaces	88
	5.2. Compact Symmetric Spaces and Explicit Form of a	
	Geodesic Diffeomorphism	89
	5.3. Explicit Computation of the Deformation Coefficient of	
	a Radial Vector Field on a Symmetric Space	92
	5.4. An Explicit Formula for the Symmetric Space Geodesic	
	Nullity	99
	5.5. Globally Minimal Surfaces of Least Volume (vol _k $X_0 = \Omega_k^0$)	00
	in Symmetric Spaces are Symmetric Spaces of Rank 1	99
	5.6. Proof of the Classification Theorem for Surfaces of Least	100
	Volume in Certain Classical Symmetric Spaces	102
Cha	ter 4. Locally Minimal Closed Surfaces Realizing Nontrivial	
		109
	Problem Formulation. Totally Geodesic Submanifolds in Lie	
310		109
	Stoups	100
§17	Necessary Results Concerning the Topological Structure	110
	. Compare 211	110
	11.1. Conomology macorae of company and a	110
	1.2. Duogioups 10tom; 1.0montose	111
	1	114
	7.4. Necessary Results Concerning Symmetric Spaces	115
§18	Lie Groups Containing a Totally Geodesic Submanifold	
J - J	Necessarily Contain Its Isometry Group	120

x Contents

§19	Realizable by Totally Geodesic Submanifolds to the Problem of the Description of (Co)Homological Properties of Cartan Models	122
§20	Classification Theorem Describing Totally Geodesic Submanifolds Realizing Nontrivial (Co)Cycles in Compact Lie Group	
	(Co)Homology	125
	20.1. The Statement of the Classification Theorem	125
	20.2. The Case of Spaces of Type II	126
	20.3. The Case of Spaces of Type I (Co)Homologically Trivial Cartan Models. Properties of the Squaring Map of a	
	Symmetric Space	127
	20.4. The Case of Spaces of Type I. Spaces $SU(k)/SO(k)$	130
	20.5. The Case of Spaces of Type I. Spaces $SU(2m)/Sp(m)$ 20.6. The Case of Spaces of Type I. Spaces $S^{2l-1} =$	135
	SO(2l)/SO(2l-1). Explicit Computation of Cocycles	
	Realizable by Totally Geodesic Submanifolds of Type I	137
	20.7. The Case of Spaces of Type I. Space E_6/F_4	139
§21	Classification Theorem Describing Cocycles in the Compact Lie	
	Group Cohomology Realizable by Totally Geodesic Spheres	145
	21.1. Classification Theorem Formulation	145
	21.2. Totally Geodesic Spheres Realizing Bott Periodicity	146
	21.3. Realization of Homotopy Group Elements of the	
	Compact Lie Groups by Totally Geodesic Spheres	148
	21.4. Necessary Results Concerning the Spinor and Semispinor	
	Representations of an Orthogonal Group	150
	21.5. Spinor Representation of the Orthogonal Group SO(8)	
	and the Cayley Number Automorphism Group	152
	21.6. Description of Totally Geodesic Spheres Realizing Nontrivial (Co)Cycles in Simple Lie Group Cohomology. The Case	
	of the Group $SU(n)$	155
	21.7. The Case of the Groups $SO(n)$ and $Sp(2n)$	156
§22	Classification Theorem Describing Elements of Homotopy Groups of Symmetric Spaces of Type I, Realizable by Totally Geodesic	
	Spheres	160
	22.1. Classification Theorem Statement	160
	22.2. Proof of the Classification Theorem. Relation between	100
	the Number of Linearly Independent Fields on Spheres	
	and that of the Elements of Homotopy Groups Realizable	
	by Totally Geodesic Spheres	162
		102
Cha	pter 5. Variational Methods for Certain Topological Problems	170
§23	Bott Periodicity from the Dirichlet Multidimensional Functional	
	Standpoint	170

Contents xi

	23.1. Explicit Description of the Bott Periodicity Isomorphism	
	for the Unitary Group	170
	23.2. Unitary Periodicity and One-Dimensional Functionals	172
	23.3. The Periodicity Theorem for a Unitary Group is Based	
	on the Dirichlet Functional Two-Dimensional Extremals	173
	23.4. The Periodicity Theorem for an Orthogonal Group is Based	
	on the 8-Dimensional Dirichlet Functional Extremals	178
00.4		181
324	Three Geometric Problems of Variational Calculus	181
	24.1. Minimal Cones and Singular Points of Minimal Surfaces	
	24.2. The Equivariant Plateau Problem	185
	24.3. Representation of Equivariant Singularities as Singular	
	Points of Closed Minimal Surfaces Embedded into	
	Symmetric Spaces	199
	24.4. On the Existence of Nonlinear Functions Whose Graphs	
	in Euclidean Space Are Minimal Surfaces	203
	24.5. Harmonic Mappings of Spheres in Nontrivial Homotopy	
	Classes	205
	24.6. A Rough Outline of Certain Recent Results on the Link of	
	Harmonic Mapping Properties to the Topology of Manifolds	210
	24.7. Properties of the Density of Smooth Mappings of	
	Manifolds	221
	24.8. The Behaviour of the Dirichlet Functional on the	
	2-Connected Manifold Diffeomorphism Group. Proof	
	of Theorem 24.6.9	225
	24.9. Necessary Topological Condition for the Existence of	
	Nontrivial Globally Minimal Harmonic Mappings	232
	24.10. The Minimization of Dirichlet-Type Functionals	239
	24.11. Regularity of Harmonic Mappings	240
		210
Cha	pter 6. Solution of the Plateau Problem in Classes of Mappings	
	pectra of Manifolds with Fixed Boundary. Construction of Globally	
Min	imal Surfaces in Variational Classes $h(A, L, L')$ and $h(A, L)$	243
825	The Cohomology Case. Computation of the Coboundary of	
320	the Pair $(X, A) = \bigcup_r (X_r, A_r)$ in Terms of Those of (X_r, A_r)	243
000		
§26	The Homology Case. Computation of the Boundary of the Pair	051
	$(X,A) = \bigcup_r (X_r, A_r)$ in Terms of the Boundaries of (X_r, A_r)	251
§27	Closedness, Invariance, and Stability of Variational Classes	257
	27.1. S-Surgery of Surfaces in a Riemannian Manifold	257
	27.2. The Closedness of Variational Classes Relative to the	
	Passage to the Limit	258
	27.3. The Invariance of Variational Classes Relative to	
	S-Surgeries of Surfaces	261
	27.4. The Stability of Variational Classes	264
000		268
§28	The General Isonerimetric Inequality	200

xii Contents

	28.1.	Choice of a Special Coordinate System	268
	28.2.	Simplicial Points of Surfaces	269
	28.3.	Isoperimetric Inequality	270
§29	The I	Minimizing Process in Variational Classes $h(A, L, L')$	
•		$g(A, ilde{L})$	277
	29.1.	The Minimizing Sequence of Surfaces. Density Functions	
		Related to Surfaces	277
		A Rough Outline of the Minimizing Process	278
	29.3.	The Constructive Method for the Minimizing Process	
		and the Proof for Its Convergence. First Step	281
		Second and Subsequent Steps in the Minimizing Process	290
	29.5.	The Theorem on the Coincidence of the Least Stratified	004
		Volume with Least λ -Vector in a Variational Class	294
§30		erties of Density Functions. The Minimality of Each Stratum	~~=
		Surface Obtained in the Minimization Process	295
	30.1.	The Value of the Density Function is Always not Less Than	905
	20.9	Unity on Each Stratum, and Unity only at Regular Points Each Stratum Is a Smooth Minimal Submanifold, Except	295
	30.2.	Possibly a Set of Singular Points of Measure Zero	304
001	ъ (
331		of Global Minimality for Constructed Stratified Surfaces	304
	31.1.	Proof of the Basic Existence Theorem for a Globally Minimal Surface	304
	21.9	The Proof of the Theorem on the Coincidence of the Least	904
	01.2.	Stratified Volume with the Least λ -Vector in a	
		Variational Class	308
622	The I	Fundamental (Co)Cycles of Globally Minimal Surfaces.	
332		Realization and Exact Spanning	309
		Fundamental (Co)Cycle Theorem	309
		Exact Minimal Realization and Exact Minimal Spanning	312
		Minimal Surfaces with Boundaries Homeomorphic to the	01-
		Sphere	314
A			
		I. Minimality Test for Lagrangian Submanifolds in Kähler Submanifolds in Kähler Manifolds. Maslov Index in	
		Surface Theory	319
		•	
	Definit		319
		ocal Minimality Test for Φ -Langrangian Submanifolds L Hermitian Manifold M^{2n}	320
		n Corollaries. New Examples of Minimal Surfaces. The	320
-		Index for Minimal Lagrangian Submanifolds	326
		tegrability Condition for the Form ψ	332
-			~~ -
		II. Calibrations, Minimal Surface Indices, Minimal Cones Codimensional and the One-Dimensional Plateau Problem	335

	Contents	xiii
Bibliography		345
Index		371