

# Contents

Preface	ix
Notes to the reader	xi
<b>1 Suffix notation and tensor algebra</b>	<b>1</b>
1.1 Summation convention	1
1.2 Free and dummy indices	3
1.3 Special symbols	4
1.4 Vector identities	6
1.5 Vector operators	8
1.6 Orthogonal coordinate systems	9
1.7 General coordinate transformations	13
1.8 Contravariant vectors	15
1.9 Kronecker delta symbol	15
1.10 Scalars and covariant vectors	16
1.11 Tensors of higher rank	18
Problems 1	19
<b>2 Special functions</b>	<b>22</b>
2.1 Origins	22
2.2 The gamma-function	23
2.3 The exponential integral and related functions	28
2.4 The error function	28
2.5 The Bessel function	31
2.6 The Bessel function $Y_\nu$	36
2.7 Generating function	39
2.8 Modified Bessel functions	42
2.9 Summary of the main properties of special functions	44
Problems 2	50
<b>3 Non-linear ordinary differential equations</b>	<b>53</b>
3.1 Introduction	53
3.2 Equations with separable variables	54

3.3	Equations reducible to linear form	57
3.4	Bernoulli's equation	59
3.5	Riccati's equation	61
3.6	Special forms of Riccati's equation	63
3.7	The Lane–Emden equation	65
3.8	The non-linear pendulum	66
3.9	Duffing's equation	68
	Problems 3	73
<b>4</b>	<b>Approximate solutions of ordinary differential equations</b>	<b>75</b>
4.1	Power series	75
4.2	Frobenius series	78
4.3	Picard iterative method	79
4.4	Perturbation series	82
4.5	Normal form	83
4.6	The W.K.B. (Wentzel–Kramers–Brillouin) approximation	84
4.7	Eigenvalue problems	88
4.8	The Liouville–Green technique	90
	Problems 4	96
<b>5</b>	<b>Contour integration</b>	<b>98</b>
5.1	Functions of a complex variable	98
5.2	Exponential and logarithmic functions	101
5.3	The derivative of a complex function	103
5.4	The Cauchy–Riemann equations	105
5.5	Derivatives of multi-valued functions	108
5.6	Complex integration	109
5.7	Cauchy's Theorem (First Integral Theorem)	112
5.8	Cauchy Integral Formula (Second Integral Theorem)	116
5.9	Derivatives of an analytic function	118
5.10	Taylor and Laurent series	120
5.11	Singularities and residues	125
5.12	Cauchy Residue Theorem	131
	Problems 5	134
<b>6</b>	<b>Applications of contour integration</b>	<b>137</b>
6.1	Introduction	137
6.2	Calculation of real integrals	137
6.3	An alternative contour	144
6.4	Poles on the real axis: the principal value of integrals	146
6.5	Branch points and integrals of many-valued functions	150
6.6	Summation of series	154
	Problems 6	158
<b>7</b>	<b>Laplace and Fourier transforms</b>	<b>161</b>
7.1	Introduction	161

7.2	The Laplace transform	162
7.3	Three basic theorems	163
7.4	The calculation of an integral	166
7.5	Laplace transform of an error function	167
7.6	Transforms of the Heaviside step function and the Dirac delta-function	169
7.7	Transforms of derivatives	171
7.8	Inversion	173
7.9	Inversions of functions with branch points	179
7.10	Solution of ordinary differential equations	181
7.11	Solution of a Volterra integral equation	182
7.12	The Fourier transform	183
	Problems 7	188
<b>8</b>	<b>Partial differential equations</b>	<b>190</b>
8.1	Introduction	190
8.2	Principle of Superposition	190
8.3	Some important equations	191
8.4	Linear second-order equations in two independent variables	193
8.5	Boundary conditions	194
8.6	Method of characteristics	195
8.7	Separation of variables	205
8.8	Integral transform techniques	214
	Problems 8	218
<b>9</b>	<b>Calculus of variations</b>	<b>224</b>
9.1	Introduction	224
9.2	Euler's equation	225
9.3	Alternative forms of Euler's equation	229
9.4	Extremal curves with discontinuous derivatives	233
9.5	Several dependent variables	233
9.6	Lagrange's equations of dynamics	234
9.7	Integrals involving derivatives higher than the first	237
9.8	Problems with constraints	238
9.9	Direct methods and eigenvalue problems	240
	Problems 9	246
	Answers to problems	248
	Index	253