

CONTENTS

1	INTRODUCTION	1
1.1	Introduction	1
1.2	Inner product spaces and topology	1
1.3	Simply connected 4-manifolds	2
1.4	Classification of intersection forms	2
1.5	Classification of topological 4-manifolds	4
1.6	Differentiable 4-manifolds	4
1.7	The existence of an exotic \mathbf{R}^4	5
1.8	Methods of proof	5
1.9	Organization	6
2	CONNECTIONS	9
2.1	Introduction	9
2.2	Sections of vector bundles	9
2.3	Connections and curvature	10
2.4	Connections and Pontrjagin classes	14
2.5	The Yang–Mills functional	16
2.6	The Yang–Mills equations	21
2.7	Connections and Chern classes	23
2.8	The gauge group	23
3	$\mathbf{SO}(3)$ -CONNECTIONS	27
3.1	Introduction	27
3.2	$\ker d^E$	28
3.3	Reducibility and isotropy groups	29
3.4	$\mathbf{SO}(2)$ -connections	32
3.5	Existence of self-dual $\mathbf{SO}(2)$ -connections	35
3.6	Inner product spaces	36
3.7	Reducible self-dual $\mathbf{SO}(3)$ -connections	38
4	THE INDEX OF THE FUNDAMENTAL COMPLEX	41
4.1	Introduction	41
4.2	Differential operators on vector bundles	41
4.3	Elliptic complexes	43
4.4	Principal G -bundles	48
4.5	The Atiyah–Singer index theorem	53

5	THE VIRTUAL MODULI SPACE $\underline{\mathcal{B}}$ OF VIRTUAL CONNECTIONS	59
5.1	Introduction	59
5.2	Virtual connections	60
5.3	Isotropy groups of the $\underline{\mathcal{G}}(E)$ action on $\underline{\mathcal{C}}(E)$	63
5.4	The slice theorem for $\underline{\mathcal{C}}$ and the topology of $\underline{\mathcal{B}}$	66
5.5	$\underline{\mathcal{B}}$ is Hausdorff	72
6	THE VIRTUAL MODULI SPACE $\underline{\mathcal{M}}$ OF VIRTUAL SELF-DUAL CONNECTIONS	73
6.1	Introduction	73
6.2	$H^*(d^E)$ as a representation of $\Gamma(d^E)$	75
6.3	The transversality theorem	77
6.4	The transversality theorem for R_-	82
6.5	Compactness of the virtual moduli space	85
6.6	Proof of Theorem 6.1.1	86
7	INTERSECTION FORMS ON 4-MANIFOLDS	89
7.1	Introduction	89
7.2	The main theorem	89
8	MODULI SPACE FOR INVARIANT SELF-DUAL VIRTUAL CONNECTIONS	93
8.1	Notation	93
8.2	The structure theorem	94
8.3	Properties of the $\underline{\mathcal{G}}^G$ action on $\underline{\mathcal{C}}^G$	95
9	APPLICATIONS TO HOMOLOGY 3-SPHERES	101
9.1	Introduction	101
9.2	Restricted manifolds and line bundles	102
9.3	Pseudofree S^1 manifolds	107
	APPENDIX A. CHARACTERISTIC CLASSES	113
A.1	Introduction	113
A.2	Projective bundles	113
A.3	Stiefel–Whitney classes	114
A.4	Chern classes	115
A.5	Pontrjagin classes	116
A.6	The Euler class	116
	APPENDIX B. $\mathbf{SO}(3)$ -VECTOR BUNDLES	119
B.1	Introduction	119
B.2	$\mathbf{SO}(3)$ -vector bundles over S^4	119
	BIBLIOGRAPHY	125

INDEX OF NOTATION

127

INDEX

129