

Table of Contents

Preface by Ronald Brown	xi
Introduction	xxi

Chapter I

Homotopy, homology, and Whitehead's classification of simply connected 4-dimensional CW-complexes 1

§0 Sufficiency, realizability and detecting functors	2
§1 Homotopy groups	5
§2 CW-complexes and homology with local coefficients	7
§3 Whitehead's certain exact sequence	10
§4 The quadratic functor Γ and central extensions	13
§5 Cup products and Pontrjagin squares	17
§6 Cohomological invariants	20
§7 A_n^2 -polyhedra and A_n^2 -forms	25
§8 Whitehead's classification of A_n^2 -polyhedra, $n \geq 2$	28

Chapter II

The CW-tower of categories 32

§1 Linear extensions of categories and exact sequences for functors	34
§2 Homotopy systems of order $(n + 1)$	44
§3 The CW-tower of categories	47
§4 The Postnikov chain functor	55
§5 Three formulas for the obstruction	68
§6 Trees of homotopy types	74
§7 On the homotopy classification of manifolds with finite fundamental group	79

Chapter III

Crossed modules and homotopy systems of order 3 87

§ 1	Nilpotent groups and Peiffer nilpotent pre-crossed modules	90
§ 2	Crossed chain complexes and homotopy systems of order 3	95
§ 3	Cylinders of CW-complexes and of crossed chain complexes	102
§ 4	Cofibrations in the category of crossed chain complexes	105
§ 5	The homotopy addition lemma	114
§ 6	A model functor from spaces to crossed chain complexes	116
§ 7	The homotopy category of 2-dimensional CW-complexes	120
§ 8	The homotopy category of 2-types	122
§ 9	The crossed chain complex of a product	126
§ 10	The action of the fundamental group and free homotopy classes	132
Appendix A. Obstructions for the realizability of chain complexes		136
Appendix B. The homotopy category of pseudo projective planes		139
Appendix C. On the suspension and the James construction		148
Appendix D. The homotopy category of suspended pseudo projective planes		155

Chapter IV

Quadratic modules and homotopy systems of order 4 166

§ 1	Quadratic modules	170
§ 2	Free quadratic modules	177
§ 3	Quadratic chain complexes	187
§ 4	Homotopies for quadratic chain maps	191
§ 5	Cofibrations in the category of quadratic chain complexes	205
§ 6	The secondary homotopy addition lemma and a model functor from spaces to quadratic chain complexes	209
§ 7	Homotopy systems of order 4	216
§ 8	The homotopy category of 3-dimensional CW-complexes	220
§ 9	The CW-tower in degree ≤ 4	226
§ 10	The homotopy category of $\bar{3}$ -types	229
§ 11	The action of the fundamental group for quadratic chain complexes	230
§ 12	The quadratic chain complex of a product	233
Appendix A. Some diverse examples and applications of quadratic chain complexes		244
Appendix B. Quadratic chain complexes and simplicial groups		258
Appendix C. Reduced and stable quadratic modules		265
Appendix D. On the homotopy classification of semi free group actions		272

Chapter V

Cohomological invariants

§ 1	The classification of 4-dimensional homotopy types	280
§ 2	A new cohomological invariant and the cup product	288
§ 3	Obstructions for the existence of certain chain maps and the primary obstruction for the realizability of a chain complex	290
§ 4	The classification of special 4-dimensional homotopy types by Pontrjagin squares	294
§ 5	Natural quotients of the Postnikov chain functor	298
§ 6	Pontrjagin squares with local coefficients	304
Appendix A. The stable equivalence classes of finite 4-dimensional complexes		311

Chapter VI

The cohomology of categories and the calculus of tracks

§ 1	The cohomology of categories	316
§ 2	The category of free $\text{nil}(2)$ -groups and the existence of Pontrjagin squares	319
§ 3	Linear track extensions of categories	329
§ 4	Free $\text{nil}(2)$ -groups and tracks for one point unions of n -spheres	339
§ 5	Tracks for one point unions of n -spheres with operators in groups	344
§ 6	Free $\text{nil}(2)$ -modules and tracks for 2-dimensional CW-complexes	348
§ 7	Track-models for 4-dimensional CW-complexes	355
Bibliography		369
List of Symbols		373
Index		377