

# Contents

---

<b>List of symbols</b>	<b>xiii</b>
<b>1 The polynomial Waring and Goldbach problems</b>	<b>1</b>
1.1 The polynomial Waring problem	2
1.2 Sums of squares	7
1.3 The polynomial Goldbach problem	12
1.4 The circle method	13
1.5 The singular series	15
<b>2 Local singular series</b>	<b>21</b>
2.1 Fundamentals	21
2.2 Characters	23
2.3 Haar measure	25
2.4 Local Radon–Nikodym derivatives	29
2.5 The case when $S(h, V)$ is empty	32
<b>3 Local Gauss sums and local derivatives</b>	<b>37</b>
3.1 A Gauss sum on $k_*$ : definition and properties	37
3.2 A modified Gauss sum	40

3.3	Computation of some special local derivatives	43
3.4	The local derivative for Waring's problem	50
<b>4</b>	<b>The adèle ring over <math>k</math></b>	<b>55</b>
4.1	Definition and first properties	55
4.2	Adèlic harmonic analysis	57
4.3	The character $E$	59
4.4	Global Radon–Nikodym derivatives	62
4.5	The idèle group of $k$	71
<b>5</b>	<b><math>L</math>-functions of Dirichlet type</b>	<b>75</b>
5.1	Multiplicative characters and $L$ -functions	75
5.2	The Riemann hypothesis	79
5.3	A brief historical note	83
5.4	A lower bound for the 3-primes singular series	84
<b>6</b>	<b>The polynomial 3-primes generating function</b>	<b>89</b>
6.1	Global Gauss sums and the summation $F_r(t)$	90
6.2	An estimate for $\sum_{\chi \in \widehat{W}}  G(\chi, t)  d\chi$	93
6.3	An asymptotic expression for $F_r(t)$ and the fundamental domains	98
<b>7</b>	<b>The polynomial 3-primes problem: an asymptotic solution</b>	<b>103</b>
7.1	Preliminary results	103
7.2	An asymptotic formula for $N(M)$	108

7.3	The 3-primes asymptotic theorem	120
<b>8</b>	<b>The polynomial Waring problem</b>	<b>123</b>
8.1	Major arc analysis	124
8.2	Minor arc analysis: Weyl's inequality for polynomials	130
8.3	The Waring singular series and the polynomial Waring theorem	137
	<b>Appendix A. A complete solution to the 3-primes problem</b>	<b>141</b>
	<b>Bibliography</b>	<b>145</b>
	<b>Index</b>	<b>155</b>