CONTENTS

PREFACE			X
SOME BASIC RE	FERENCES		/ X
NOTATIONS			xvii
CHAPTER I.	INTRODUCT	TION	1
1	. Introdu	action	1
	2. Prope	erties of Polynomials	1
	2.1.	<u>-</u>	1
	2.2.	Some Special Polynomials	4
	3. Eleme	entary Inequalities	5
	3.1.	Bernoulli's Inequality	5
	3.2.	Inequalities Involving Some Elementary	
		Functions	6
	_	erties of Sequences	8
	4.1.	Convex Sequences; Sequences of Bounded	
		Variation	8
		Logarithmically Convex Sequences	13
		An Order Relation for Sequences	18
		ex Functions	21
		Convex Functions of a Single Variable	21
		Convex Functions of Several Variables	31
	5.3.	Higher Order Convexity	32
CHAPTER II.	THE ARITH	METIC, GEOMETRIC AND HARMONIC MEANS	34
	1. Defin	nitions and Simple Properties	34
	1.1.	The Arithmetic Mean	34
	1.2.	The Geometrical and Harmonic Means	36
	1.3.	Some Interpretations and Applications	38
		Geometric-Arithmetic Mean Inequality	42
		The Statement of the Theorem	42
		Some Preliminary Results	43
		Some Geometric Interpretations	53
		Proofs of GA	56
	2.5.		90
		nements of the Geometric-Arithmetic	
		an Inequality	94
	-	The Inequalities of Rado and Popoviciu	94
	3.2.	Extensions of the Inequalities of Rado	00
	2 2	and Popoviciu	99
	3.3.		105
	3.4.		109
	3.5. 3.6.		113 117
		Oriter Verritements	11/

viii . CONTENTS

		4. Converse Inequalities	122
		4.1. The Difference An-Gn	122
		4.2. The Ratio A _n /G _n " "	124
		5. Some Miscellaneous Results	126
CHAPTER	TTT.	THE POWER MEANS	132
CHAPTER	111.	1. Definition and Simple Properties	132
		2. Sums of Power	136
		2.1. Hölder's Inequality	136
		2.2. Minkowski's Inequality	147
		2.3. Refinements of the	
		Hölder and Minkowski Inequalities	149
		3. Deeper Properties of the Power Means	159
		3.1. The Fundamental Inequality	159
		3.2. Refinements of (r;s)	172
		4. Generalizations of the Power Means	185
		4.1. Counter-Harmonic Means	185
		4.2. Some Means Due of Gini and Bonferroni	189
		4.3. Mixed Means	191
		5. Converse Inequalities	194
		5.1. Ratios of Power Means	195
		5.2. Differences of Power Means	204
		5.3. Converse Cauchy, Hölder and	0.07
		Minkowski Inequalities	207
		6. Some Odd Results	212
CHAPTER	IV.	THE QUASI-ARITHMETIC MEANS	215
•		 Definition and Simple Properties 	215
		1.1. The Definition	215
		1.2. Some Examples	218
		1.3. Equivalent Quasi-Arithmetic Means	220
		2. Comparable Means	224
		 Results of Rado, Popoviciu and Everitt Type 	234
		3.1. Some General Inequalities	234
		3.2. Some Applications of the	227
		General Inequalities	237 242
		4. Chakalov's Inequality	242
		Generalizations of the Hölder	245
		and Minkowski Inequalities of	
		6. Converse Inequalities	256
		7. Generalizatins of the Quasi-Arithmetic Means	261
		7.1. A Theorem of Losonczi	261
		7.2. Further Generalizations	269
		8. Some Further Inequalities	271 271
		8.1. A Theorem of Godunova	271
		8.2. A Problem of Oppenheim	2/3
		G.Z. R IIODZCM OI OFF	270
		8.3. An Inequality Due to Ky Fan 8.4. Means on the Move	279 282

CONTENTS		

ix

451

CHAPTER V.	SYMMETRIC MEANS	283
	 Definitions and Simple Properties 	283
	 Relations Between the Elementary 	
	Symmetric Functions and Means	285
	3. Inequalities of the Rado-Popoviciu Type	299
	4. The Inequalities of Marcus and Lopes	306
	5. Generalizations of the Symmetric Means	310
	5.1. Symmetric Means as Mixed Means	310
	5.2. The Symmetric Means of Hamy	311
	5.3. The Complete Symmetric Mean	313
	5.4. The Biplanar Means	316
	5.5. The Means of Whiteley	317
	5.6. Some Forms of Whiteley	324
	5.7. Muirhead Means	333
CHAPTER VI	FURTHER MEANS, AXIOMATICS AND OTHER TOPICS	343
CHAPTER VI	1. Introduction	343
	2. Mean-Value Means	343
	3. The Logarithmic Mean and Generalizations	345
	4. Level Surface Means	351
	5. Corresponding Means	353
	6. A Mean of Galvani	354
	7. Means and Graphs	354
	8. The Admissible Means of Bauer	356
	9. Compounding of Means	359
	10. The Hypergeometric Mean	368
	11. The Almost Symmetric Functions of Segre	369
	12. Axiomatization of Means	372
	13. Hlawka-Type Inequalities	375
	14. Integral Means: Basic Results	377
	15. Integral Means: Further Results	382
BIBLIOGRAPHY	YY	387

AUTHOR INDEX