

Contents

Translator's Preface	viii
Introduction to the English Edition	ix
Introduction to the Second Russian Edition	x
Notation	xii
Chapter I. Integer Points	1
§1. Statement of the Problem, Auxiliary Remarks, and the Simplest Results	1
§2. The Connection Between Problems in the Theory of Integer Points and Trigonometric Sums	6
§3. Theorems on Trigonometric Sums	10
§4. Integer Points in a Circle and Under a Hyperbola	21
Exercises	25
Chapter II. Entire Functions of Finite Order	27
§1. Infinite Products. Weierstrass's Formula	27
§2. Entire Functions of Finite Order	32
Exercises	38
Chapter III. The Euler Gamma Function	41
§1. Definition and Simplest Properties	41
§2. Stirling's Formula	44
§3. The Euler Beta Function and Dirichlet's Integral	45
Exercises	48
Chapter IV. The Riemann Zeta Function	51
§1. Definition and Simplest Properties	51
§2. Simplest Theorems on the Zeros	56
§3. Approximation by a Finite Sum	61
Exercises	62
Chapter V. The Connection Between the Sum of the Coefficients of a Dirichlet Series and the Function Defined by this Series	64
§1. A General Theorem	64
§2. The Prime Number Theorem	66

§3. Representation of the Chebyshev Functions as Sums Over the Zeros of the Zeta Function	69
Exercises	70
Chapter VI. The Method of I.M. Vinogradov in the Theory of the Zeta Function	73
§1. Theorem on the Mean Value of the Modulus of a Trigonometric Sum	73
§2. Estimate of a Zeta Sum	82
§3. Estimate for the Zeta Function Close to the Line $\sigma = 1$	86
§4. A Function-Theoretic Lemma	87
§5. A New Boundary for the Zeros of the Zeta Function.	88
§6. A New Remainder Term in the Prime Number Theorem.	90
Exercises	91
Chapter VII. The Density of the Zeros of the Zeta Function and the Problem of the Distribution of Prime Numbers in Short Intervals	94
§1. The Simplest Density Theorem.	94
§2. Prime Numbers in Short Intervals	98
Exercises	100
Chapter VIII. Dirichlet L-Functions	102
§1. Characters and their Properties	102
§2. Definition of L-Functions and their Simplest Properties	110
§3. The Functional Equation	113
§4. Non-trivial Zeros; Expansion of the Logarithmic Derivative as a Series in the Zeros.	116
§5. Simplest Theorems on the Zeros	117
Exercises	119
Chapter IX. Prime Numbers in Arithmetic Progressions	122
§1. An Explicit Formula	122
§2. Theorems on the Boundary of the Zeros	124
§3. The Prime Number Theorem for Arithmetic Progressions	135
Exercises	138
Chapter X. The Goldbach Conjecture	141
§1. Auxiliary Statements.	141
§2. The Circle Method for Goldbach's Problem.	142
§3. Linear Trigonometric Sums with Prime Numbers	149
§4. An Effective Theorem	153
Exercises	158

Chapter XI. Waring's Problem	160
§1. The Circle Method for Waring's Problem	160
§2. An Estimate for Weyl Sums and the Asymptotic Formula for Waring's Problem	171
§3. An Estimate for $G(n)$	174
Exercises	177
Hints for the Solution of the Exercises	181
Table of Prime Numbers < 4070 and their Smallest Primitive Roots ..	217
Bibliography	219
Subject Index	221