

Table of Contents

1.	Introduction	1
1.1	Introduction	1
	History	1
	Recent Advances	8
1.2	Definitions and Examples	10
	Examples	11
	Basic Constructions	13
	The Module of \mathcal{A} -Derivations	15
	The Complement of a Complex Arrangement	16
	Reflection Arrangements	16
1.3	Outline	17
	Combinatorics	17
	Algebras	17
	The Module of \mathcal{A} -Derivations	18
	Topology	19
	Reflection Arrangements	20
2.	Combinatorics	23
2.1	The Poset $L(\mathcal{A})$	23
	Definitions	23
	Examples	26
	Oriented Matroids	28
	Supersolvable Arrangements	30
2.2	The Möbius Function	32
	The Möbius Function	32
	Möbius Inversion	34
	The Function $\mu(X)$	35
	Historical Notes	38
2.3	The Poincaré Polynomial	42
	Examples	42
	The Deletion–Restriction Theorem	46
	Supersolvable Arrangements	48
	Nice Partitions	50
	Counting Functions	51

2.4	Graphic Arrangements	52
	Definitions	52
	Deletion–Contraction	54
	Acyclic Orientations	57
3.	Algebras	59
3.1	$A(\mathcal{A})$ for Central Arrangements	60
	Construction of $A(\mathcal{A})$	60
	An Acyclic Complex	62
	The Structure of $A(\mathcal{A})$	63
	The Injective Map $A(\mathcal{A}_X) \rightarrow A(\mathcal{A})$	65
	The Broken Circuit Basis	67
3.2	$A(\mathcal{A})$ for Affine Arrangements	70
	Construction of $A(\mathcal{A})$	70
	The Broken Circuit Basis	72
	Deletion and Restriction	74
	The Structure of $A(\mathcal{A})$	77
	A -equivalence	78
3.3	Algebra Factorizations	79
	Supersolvable Arrangements	80
	Nice Partitions of Central Arrangements	82
	Nice Partitions of Affine Arrangements	85
3.4	The Algebra $B(\mathcal{A})$	86
	The Shuffle Product	86
	The Algebra $B(\mathcal{A})$	88
	The Isomorphism of B and A	89
3.5	Differential Forms	92
	The de Rham Complex	92
	The Algebra $R(\mathcal{A})$	93
	Deletion and Restriction	95
	The Isomorphism of R and A	97
4.	Free Arrangements	99
4.1	The Module $D(\mathcal{A})$	100
	Derivations	100
	Basic Properties	102
4.2	Free Arrangements	104
	Saito’s Criterion	104
	Exponents	107
	Examples	111
4.3	The Addition–Deletion Theorem	113
	Basis Extension	114
	The Map from $D(\mathcal{A})$ to $D(\mathcal{A}'')$	115
	The Addition–Deletion Theorem	117
	Inductively Free Arrangements	119
	Supersolvable Arrangements	121

Factorization Theorem	122
4.4 The Modules $\Omega^p(\mathcal{A})$	123
Definition of $\Omega^p(\mathcal{A})$	124
Basic Properties of $\Omega^p(\mathcal{A})$	124
The Acyclic Complex $(\Omega(\mathcal{A}), \partial)$	133
The η -Complex $(\Omega(\mathcal{A}), \partial_h)$	133
4.5 Lattice Homology	135
The Order Complex	136
The Folkman Complex	137
The Homology Groups	140
The Homotopy Type	141
Whitney Homology	142
Connection with the Folkman Complex	144
4.6 The Characteristic Polynomial	145
The Order Complex with Functors	145
Local Functors	147
The Homology $H_p(\mathcal{A}, F)$	148
The Polynomial $\Psi(\mathcal{A}, x, t)$	150
The Factorization Theorem	154
5. Topology	157
5.1 The Complement $M(\mathcal{A})$	158
$K(\pi, 1)$ -Arrangements	159
Free Arrangements	163
Generic Arrangements	164
Deformation	166
Arnold's Conjectures	167
5.2 The Homotopy Type of $M(\mathcal{A})$	168
Real Arrangements	168
The Homotopy Type	171
Complexified Real Arrangements	173
Salvetti's Complex	175
The Homotopy Equivalence	176
5.3 The Fundamental Group	177
Admissible Graphs	179
Arvola's Presentation	184
5.4 The Cohomology of $M(\mathcal{A})$	190
The Thom Isomorphism	191
Brieskorn's Lemma	195
5.5 The Fibration Theorem	196
Horizontal Subspaces	197
Good Subspaces	198
Good Lines	199
5.6 Related Research	202
Minimal Models	202

Discriminantal Arrangements	205
Alexander Duality	207
The Milnor Fiber of a Generic Arrangement	209
Arrangements of Subspaces	211
6. Reflection Arrangements	215
6.1 Equivariant Theory	216
The Action of G	216
Matrices	218
Character Formulas	219
Topological Interpretation	222
6.2 Reflection Arrangements	223
Basic Properties	223
Examples	225
Relative Invariants	228
Jacobian and Discriminant	229
Classification	231
6.3 Free Arrangements	232
Invariant Theory	232
The Hessian	234
$D_R(\delta)$ Is Free	235
$D(\mathcal{A})$ Is Free	237
The Discriminant Matrix	238
A Character Formula	241
6.4 The Structure of $L(\mathcal{A})$	243
The Symmetric Group	243
The Full Monomial Group	244
The Monomial Group $G(r, r, \ell)$	247
The Exceptional Groups	251
6.5 Restrictions	254
The Cardinality of \mathcal{A}^H	254
\mathcal{A}^H Is Free In Coxeter Arrangements	256
6.6 Topology	259
Stratification of the Discriminant	259
Shephard Groups	265
The $K(\pi, 1)$ Problem	267
A. Some Commutative Algebra	271
A.1 Free Modules	271
A.2 Krull Dimension	272
A.3 Graded Modules	274
A.4 Associated Primes and Regular Sequences	276

B. Basic Derivations	279
B.1 The Infinite Families	279
B.2 Exceptional Groups of Rank 2	280
B.3 Exceptional Groups of Rank ≥ 3	280
B.4 The Coexponents	286
C. Orbit Types	289
D. Three-Dimensional Restrictions	301
References	303
Index	315
Index of Symbols	323