## **Contents**

Introduction		Xi
1	Notational Conventions	xv
<b>I.</b> 3	Differentiable Structures	1
1.	Smooth Manifolds and Maps	1
2.	Partitions of Unity	6
3.	Smooth Vector Bundles	8
4.	Tangent Space	12
5.	Vector Fields	16
6.	Differential Equations on a Smooth Manifold	18
7.	Collars	21
II.	Immersions, Imbeddings, Submanifolds	25
1.	Local Equivalence of Maps	25
2.	Submanifolds	26
3.	Imbeddings in $\mathbb{R}^n$	32
4.	Isotopies	33
5.	Ambient Isotopies	36
6.	Historical Remarks	38
III.	Normal Bundle, Tubular Neighborhoods	41
1.	Exponential Map	41
2.	Normal Bundle and Tubular Neighborhoods	44

iii	CONTENTS

3. Uniqueness of Tubular Neighborhoods	49
4. Submanifolds of the Boundary	52
5. Inverse Image of a Regular Value	55
6. The group $\Gamma^m$	56
7. Remarks	57
IV. Transversality	59
1. Transversal Maps and Manifolds	59
2. Transversality Theorem	63
3. Morse Functions	66
4. Neighborhood of a Critical Point	68
5. Intersection Numbers	70
6. Historical Remarks	73
V. Foliations	75
1. d-Fields	76
2. Foliations	78
3. Frobenius Theorem	80
4. Leaves of a Foliation	82
5. Examples	84
VI. Operations on Manifolds	89
1. Connected Sum	90
2. # and Homotopy Spheres	94
3. Boundary Connected Sum	97
4. Joining Manifolds along Submanifolds	99
5. Joining Manifolds along Submanifolds of the Boundary	
6. Attaching Handles	103
7. Cancellation Lemma	106
8. Combinatorial Attachment	110
9. Surgery	112
10. Homology and Intersections in a Handle	113
11. $(m, k)$ -Handlebodies, $m > 2k$	115
12. (2k, k)-Handlebodies; Plumbing	118
VII. Handle Presentation Theorem	125
1. Elementary Cobordisms	125
2. Handle Presentation Theorem	127
3. Homology Data of a Cobordism	131
4. Morse Inequalities	135
5. Poincaré Duality	136
6. 0-Dimensional Handles	137
7. Heegaard Diagrams	138
8. Historical Remarks	141

CONTENTS			i

VIII. The h-Cobordism Theorem	14.
1. Elementary Row Operations	144
2. Cancellation of Handles	14
3. 1-Handles	15
4. Minimal Presentation; Main Theorems	152
5. h-cobordism; The Group $\theta^m$	150
6. Highly Connected Manifolds	159
7. Remarks	16
IX. Framed Manifolds	16′
1. Framings	16
2. Framed Submanifolds	17
3. $\Omega^k(M^m)$	174
$4.  \Omega^0(M^m)$	17
5. The Pontriagin Construction	179
6. Operations on Framed Submanifolds and Homotopy Theory	183
7. π-Manifolds	180
8. Almost Parallelizable Manifolds	189
9. Historical Remarks	192
X. Surgery	19:
1. Effect of Surgery on Homology	19
2. Framing a Surgery; Surgery below Middle Dimension	200
3. Surgery on 4n-Dimensional Manifolds	202
4. Surgery on $(4n + 2)$ -Dimensional Manifolds	20
5. Surgery on Odd-Dimensional Manifolds	210
6. Computation of $\theta^n$	21:
7. Historical Note	219
Appendix	223
1. Implicit Function Theorem	223
2. A Lemma of M. Morse	220
3. Brown-Sard Theorem	220
4. Orthonormalization	22
5. Homotopy Groups of $SO(k)$	230
Bibliography	23
Index	24