

Contents of Minimal Surfaces II

Introduction	1
Part III. Boundary Behaviour of Minimal Surfaces	
Chapter 7. The Boundary Regularity of Minimal Surfaces	6
7.1 Potential-Theoretic Preparations	7
7.2 Solutions of Differential Inequalities	21
7.3 The Boundary Regularity of Minimal Surfaces Bounded by Jordan Arcs	33
7.4 The Boundary Behaviour of Minimal Surfaces at Their Free Boundary: A Survey of the Results and an Outline of Their Proofs ..	43
7.5 Hölder Continuity for Minima	48
7.6 Hölder Continuity for Stationary Surfaces	60
7.7 $C^{1,1/2}$ -Regularity	83
7.8 Higher Regularity in Case of Support Surfaces with Empty Boundaries. Analytic Continuation Across a Free Boundary	102
7.9 A Different Approach to Boundary Regularity	109
7.10 Asymptotic Expansion of Minimal Surfaces at Boundary Branch Points and Geometric Consequences	117
7.11 The Gauss-Bonnet Formula for Branched Minimal Surfaces	121
7.12 Scholia	128
Chapter 8. Singular Boundary Points of Minimal Surfaces	141
8.1 The Method of Hartman and Wintner, and Asymptotic Expansions at Boundary Branch Points	142
8.2 A Gradient Estimate at Singularities Corresponding to Corners of the Boundary	163
8.3 Minimal Surfaces with Piecewise Smooth Boundary Curves and Their Asymptotic Behaviour at Corners	173
8.4 An Asymptotic Expansion for Solutions of the Partially Free Boundary Problem	186
8.5 Scholia	196

Chapter 9. Minimal Surfaces with Supporting Half-Planes	198
9.1 An Experiment	199
9.2 Examples of Minimal Surfaces with Cusps on the Supporting Surface	202
9.3 Set-up of the Problem. Properties of Stationary Solutions	206
9.4 Classification of the Contact Sets	208
9.5 Nonparametric Representation, Uniqueness, and Symmetry of Solutions	213
9.6 Asymptotic Expansions for Surfaces of Cusp-Types I and III. Minima of Dirichlet's Integral	216
9.7 Asymptotic Expansions for Surfaces of the Tongue/Loop-Type II ..	218
9.8 Final Results on the Shape of the Trace. Absence of Cusps. Optimal Boundary Regularity	221
9.9 Proof of the Representation Theorem	223
9.10 Scholia	229
1. Remarks about Chapter 9. 2. Numerical Solutions. 3. Another Uniqueness Theorem for Minimal Surfaces with a Semifree Boundary.	

Part IV. Ramifications: The Thread Problem. The General Plateau Problem

Chapter 10. The Thread Problem	250
10.1 Experiments and Examples. Mathematical Formulation of the Simplest Thread Problem	250
10.2 Existence of Solutions to the Thread Problem	255
10.3 Analyticity of the Movable Boundary	271
10.4 Scholia	291
Chapter 11. The General Problem of Plateau	293
11.1 The General Problem of Plateau. Formulation and Examples	293
11.2 A Geometric Approach to Teichmüller Theory of Oriented Surfaces	299
11.3 Symmetric Riemann Surfaces and Their Teichmüller Spaces	307
11.4 The Mumford Compactness Theorem	315
11.5 The Variational Problem	319
11.6 Existence Results for the General Problem of Plateau in \mathbb{R}^3	328
11.7 Scholia	339
Bibliography	341
Index of Names	397
Subject Index	400
Index of Illustrations	
Minimal Surfaces II	415
Minimal Surfaces I	417
Sources of Illustrations of Minimal Surfaces II	422

Contents of Minimal Surfaces I

Introduction	1
Part I. Introduction to the Geometry of Surfaces and to Minimal Surfaces	
Chapter 1. Differential Geometry of Surfaces in Three-Dimensional Euclidean Space	6
1.1 Surfaces in Euclidean Space	7
1.2 Gauss Map, Weingarten Map. First, Second, and Third Fundamental Form. Mean Curvature and Gauss Curvature	11
1.3 Gauss's Representation Formula, Christoffel Symbols, Gauss-Codazzi Equations, Theorema Egregium, Minding's Formula for the Geodesic Curvature	25
1.4 Conformal Parameters. Gauss-Bonnet Theorem	34
1.5 Covariant Differentiation. The Beltrami Operator	40
1.6 Scholia	48
1. Textbooks. 2. Annotations to the History of the Theory of Surfaces. 3. References to the Sources of Differential Geometry and to the Literature on Its History.	
Chapter 2. Minimal Surfaces	53
2.1 First Variation of Area. Minimal Surfaces	54
2.2 Nonparametric Minimal Surfaces	58
2.3 Conformal Representation and Analyticity of Nonparametric Minimal Surfaces	61
2.4 Bernstein's Theorem	65
2.5 Two Characterizations of Minimal Surfaces	71
2.6 Parametric Surfaces in Conformal Parameters. Conformal Representation of Minimal Surfaces. General Definition of Minimal Surfaces	74
2.7 A Formula for the Mean Curvature	77
2.8 Absolute and Relative Minima of Area	80
2.9 Scholia	85
1. References to the Literature on Nonparametric Minimal Surfaces. 2. Bernstein's Theorem. 3. Stable Minimal Surfaces. 4. Foliations by Minimal Surfaces.	

Chapter 3. Representation Formulas and Examples of Minimal Surfaces	89
3.1 The Adjoint Surface. Minimal Surfaces as Isotropic Curves in \mathbb{C}^3 . Associate Minimal Surfaces	90
3.2 Behaviour of Minimal Surfaces Near Branch Points	101
3.3 Representation Formulas for Minimal Surfaces	107
3.4 Björling's Problem. Straight Lines and Planar Lines of Curvature on Minimal Surfaces. Schwarzian Chains	120
3.5 Examples of Minimal Surfaces	135
1. Catenoid and Helicoid. 2. Scherk's Second Surface: The General Minimal Surface of Helicoidal Type. 3. The Enneper Surface. 4. Bour Surfaces. 5. Thomsen Surfaces. 6. Scherk's First Surface. 7. The Henneberg Surface. 8. Catalan's Surface. 9. Schwarz's Surface.	
3.6 Complete Minimal Surfaces	175
3.7 Omissions of the Gauss Map of Complete Minimal Surfaces	181
3.8 Scholia	192
1. Historical Remarks and References to the Literature. 2. Complete Minimal Surfaces of Finite Total Curvature and of Finite Topology. 3. Complete Properly Immersed Minimal Surfaces. 4. Construction of Minimal Surfaces. 5. Triply Periodic Minimal Surfaces.	

Part II. Plateau's Problem and Free Boundary Problems

Chapter 4. The Plateau Problem and the Partially Free Boundary Problem for Minimal Surfaces	221
4.1 Area Functional Versus Dirichlet Integral	226
4.2 Rigorous Formulation of Plateau's Problem and of the Minimization Process	231
4.3 Existence Proof, Part I: Solution of the Variational Problem	234
4.4 The Courant-Lebesgue Lemma	239
4.5 Existence Proof, Part II: Conformality of Minimizers of the Dirichlet Integral	242
4.6 Variant of the Existence Proof. The Partially Free Boundary Problem	253
4.7 Boundary Behaviour of Minimal Surfaces with Rectifiable Boundaries	259
4.8 Reflection Principles	267
4.9 Uniqueness and Nonuniqueness Questions	270
4.10 Scholia	276
1. Historical Remarks and References to the Literature. 2. Branch Points. 3. Embedded Solutions of Plateau's Problem. 4. More on Uniqueness and Nonuniqueness. 5. Index Theorems, Generic Finiteness, and Morse-Theory Results. 6. Obstacle Problems. 7. Systems of Minimal Surfaces.	

Chapter 5. Minimal Surfaces with Free Boundaries	303
5.1 Surfaces of Class H_2^1 and Homotopy Classes of Their Boundary Curves. Nonsolvability of the Free Boundary Problem with Fixed Homotopy Type of the Boundary Traces	305
5.2 Classes of Admissible Functions. Linking Condition	318
5.3 Existence of Minimizers for the Free Boundary Problem	321
5.4 Stationary Minimal Surfaces with Free or Partially Free Boundaries and the Transversality Condition	328
5.5 Necessary Conditions for Stationary Minimal Surfaces	335
5.6 Existence of Stationary Minimal Surfaces in a Simplex	339
5.7 Stationary Minimal Surfaces of Disk-Type in a Sphere	341
5.8 Report on the Existence of Stationary Minimal Surfaces in Convex Bodies	343
5.9 Nonuniqueness of Solutions to a Free Boundary Problem. Families of Solutions	345
5.10 Scholia	365
 Chapter 6. Enclosure Theorems and Isoperimetric Inequalities for Minimal Surfaces with Fixed or Free Boundaries	 367
6.1 Applications of the Maximum Principle and Nonexistence of Multiply Connected Minimal Surfaces with Prescribed Boundaries ..	368
6.2 Touching H-Surfaces and Enclosure Theorems. Further Nonexistence Results	372
6.3 Isoperimetric Inequalities	382
6.4 Estimates for the Length of the Free Trace	396
6.5 Scholia	420
1. The Isoperimetric Problem. Historical Remarks and References to the Literature.	
2. Experimental Proof of the Isoperimetric Inequality. 3. Estimates for the Length of the Free Trace. 4. Enclosure Theorems and Nonexistence.	
 Bibliography	 427
Index of Names	483
Subject Index	486
 Index of Illustrations	
Minimal Surfaces I	501
Minimal Surfaces II	506
Sources of Illustrations of Minimal Surfaces I	508
Colour Plates I–VIII	after page 218