

Contents

Preface	xi
Acknowledgments	xv
Notation	xvii
Chapter I Tools from Number Theory	1
I.1. Partial Summation	2
I.2. Arithmetical Functions, Convolution, Möbius Inversion Formula	4
I.3. Periodic Functions, Even Functions, Ramanujan Sums	15
I.4. The Turán–Kubilius Inequality	19
I.5. Generating Functions, Dirichlet Series	25
I.6. Some Results on Prime Numbers	31
I.7. Characters, L-Functions, Primes in Arithmetic Progressions	35
I.8. Exercises	39
Photographs	43
Chapter II Mean-Value Theorems and Multiplicative Functions, I	45
II.1. Motivation	46
II.2. Elementary Mean-Value Theorems (Wintner, Axer)	49
II.3. Estimates for Sums over Multiplicative Functions (Rankin's Trick)	56
II.4. Wirsing's Mean-Value Theorem for Sums over Non-Negative Multiplicative Functions	65
II.5. The Theorem of G. Halász on Mean-Values of Complex-Valued Multiplicative Functions	76
II.6. The Theorem of Daboussi and Delange on the Fourier-Coefficients of Multiplicative Functions	78
II.7. Application of the Daboussi–Delange Theorem to a Problem of Uniform Distribution	81
II.8. The Theorem of Saffari and Daboussi, I.	82
II.9. Daboussi's Elementary Proof of the Prime Number Theorem	85
II.10. Mohan Nair's Elementary Method in Prime Number Theory	91

II.11. Exercises	93
Chapter III Related Arithmetical Functions	97
III.1. Introduction, Motivation	98
III.2. Main Results	101
III.3. Lemmata, Proof of Theorem 2.3	104
III.4. Applications	110
III.5. On a Theorem of L. Lucht	115
III.6. The Theorem of Saffari and Daboussi, II	117
III.7. Application to Almost-Periodic Functions	118
III.8. Exercises	121
Chapter IV Uniformly Almost-Periodic Arithmetical Functions	123
IV.1. Even and Periodic Arithmetical Functions	124
IV.2. Simple Properties	133
IV.3. Limit Distributions	139
IV.4. Gelfand's Theory: Maximal Ideal Spaces	142
IV.4.A. The maximal ideal space $\Delta_{\mathcal{B}}$ of \mathcal{B}^u	142
IV.4.B. The maximal ideal space $\Delta_{\mathcal{D}}$ of \mathcal{D}^u	147
IV.5. Application of Tietze's Extension Theorem	155
IV.6. Integration of Uniformly Almost-Even Functions	156
IV.7. Exercises	162
Chapter V Ramanujan Expansions of Functions in \mathcal{B}^u	165
V.1. Introduction	166
V.2. Equivalence of Theorems 1.2, 1.3, 1.4, 1.5	168
V.3. Some Lemmata	171
V.4. Proof of Theorem 1.5	175
V.5. Proof of Lemmas 3.4 and 3.5	178
V.6. Exercises	184
Chapter VI Almost-Periodic and Almost-Even Arithmetical Functions	185
VI.1. Besicovich Norm, Spaces of Almost Periodic Functions	186
VI.2. Some Properties of Spaces of q-Almost-Periodic Functions	197

Contents

VI.3. Parseval's Equation	206
VI.4. A Second Proof for Parseval's Formula	208
VI.5. An Approximation for Functions in \mathcal{B}^1	210
VI.6. Limit Distributions of Arithmetical Functions	212
VI.7. Arithmetical Applications	215
VI.7.A. Mean-Values, Limit Distributions	215
VI.7.B. Applications to Power-Series with Multiplicative Coefficients	218
VI.7.C. Power Series Bounded on the Negative Real Axis	221
VI.8. A \mathcal{B}^q -Criterion	224
VI.9. Exercises	229
Photographs	231
 Chapter VII The Theorems of Elliott and Daboussi	233
VII.1. Introduction	234
VII.2. Multiplicative Functions with Mean-Value $M(f) \neq 0$, Satisfying $\ f\ _2 < \infty$	239
VII.3. Criteria for Multiplicative Functions to Belong to \mathcal{B}^1	243
VII.4. Criteria for Multiplicative Functions to Belong to \mathcal{B}^q	251
VII.5. Multiplicative Functions in \mathcal{A}^q with Mean-Value $M(f) \neq 0$	257
VII.6. Multiplicative Functions in \mathcal{A}^q with Non-Void Spectrum	261
VII.7. Exercises	266
 Chapter VIII Ramanujan Expansions	269
VIII.1. Introduction	270
VIII.2. Wintner's Criterion	271
VIII.3. Mean-Value Formulae for Multiplicative Functions	276
VIII.4. Formulae for Ramanujan Coefficients	280
VIII.5. Pointwise Convergence of Ramanujan Expansions	284
VIII.6. Still Another Proof for Parseval's Equation	289
VIII.7. Additive Functions	291
VIII.8. Exercises	291

Chapter IX Mean-Value Theorems and Multiplicative Functions, II	293
IX.1. On Wirsing's Mean-Value Theorem	294
IX.2. Proof of Theorem 1.4	298
IX.3. The Mean-Value Theorem of Gabor Halász	303
IX.4. Proof of Proposition 3.3	309
IX.5. Exercises	311
Photographs	313
Appendix	315
A.1. The Stone-Weierstrass Theorem, Tietze's Theorem	315
A.2. Elementary Theory of Hilbert Space	316
A.3. Integration	319
A.4. Tauberian Theorems (Hardy-Littlewood-Karamata, Landau-Ikehara)	321
A.5. The Continuity Theorem for Characteristic Functions	323
A.6. Gelfand's Theory of Commutative Banach Algebras	325
A.7. Infinite Products	327
A.8. The Large Sieve	329
A.9. Dirichlet Series	331
Bibliography	333
Author Index	353
Subject Index	357
Photographs	365
Acknowledgements	367