

Contents

I	The General Idea Behind Gödel's Proof	1
I.	Abstract Forms of Gödel's and Tarski's Theorems . . .	5
II.	Undecidable Sentences of \mathcal{L}	10
II	Tarski's Theorem for Arithmetic	14
I.	The Language \mathcal{L}_E	14
§1.	Syntactic Preliminaries.	14
§2.	The Notion of Truth in \mathcal{L}_E	17
§3.	Arithmetic and arithmetic Sets and Relations.	19
II.	Concatenation and Gödel Numbering	20
§4.	Concatenation to the Base b	20
§5.	Gödel Numbering.	22
III.	Tarski's Theorem	24
§6.	Diagonalization and Gödel Sentences.	24
III	The Incompleteness of Peano Arithmetic With Ex-	
	ponentiation	28
I.	The Axiom System P.E.	28
§1.	The Axiom System P.E.	28
II.	Arithmetization of the Axiom System	30
§2.	Preliminaries.	30
§3.	Arithmetization of the Syntax of P.E.	33
§4.	Gödel's Incompleteness Theorem for P. E.. . . .	36
IV	Arithmetic Without the Exponential	40
I.	The Incompleteness of P.A.	40
§3.	Concatenation to a Prime Base.	43
§4.	The Finite Set Lemma.	45
§5.	Proof of Theorem E.	46
§6.	The Incompleteness of Peano Arithmetic.	49
II.	More on Σ_1 -Relations	50

V	Gödel's Proof Based on ω-Consistency	56
I.	Some Abstract Incompleteness Theorems	58
	§1. A Basic Incompleteness Theorem.	58
	§2. The ω -Consistency Lemma.	61
II.	Σ_0 -Completeness	66
	§4. Some Σ_0 -complete Subsystems of Peano Arith- metic.	68
	§6. The ω -Incompleteness of P.A.	73
VI	Rosser Systems	75
	§1. Some Abstract Incompleteness Theorems Af- ter Rosser.	76
	§2. A General Separation Principle.	77
	§3. Rosser's Undecidable Sentence.	81
	§4. The Gödel and Rosser Sentences Compared. . .	82
	§5. More on Separation.	84
VII	Shepherdson's Representation Theorems	86
	§1. Shepherdson's Representation Theorem.	86
	§2. Exact Rosser Systems.	90
	§3. Some Variants of Rosser's Undecidable Sentence. .	93
	§4. A Strengthening of Shepherdson's Theorems .	96
VIII	Definability and Diagonalization	97
	§1. Definability and Complete Representability. . .	97
	§2. Strong Definability of Functions in \mathcal{S}	98
	§3. Strong Definability of Recursive Functions in (R).	100
	§4. Fixed Points and Gödel Sentences.	102
	§5. Truth Predicates.	104
IX	The Unprovability of Consistency	106
	§1. Provability Predicates.	106
	§2. The Unprovability of Consistency.	108
	§3. Henkin Sentences and Löb's Theorem.	109
X	Some General Remarks on Provability and Truth	112
XI	Self-Referential Systems	116
I.	Logicians Who Reason About Themselves	116
	§1. An Analogue of the Tarski-Gödel Theorem. . .	116
	§2. Normal and Stable Reasoners of Type 1.	117
	§3. Rosser Type Reasoners.	120

§4.	The Consistency Problem.	122
§5.	Self-Fulfilling Beliefs and Löb's Theorem.	124
II.	Incompleteness Arguments in a General Setting	126
III.	Systems of Type G	129
IV.	Modal Systems	132
	References	136
	Index	138