Contents

Suggestions to the reader		xi
Chap	oter 1. Algebraic number theory	1
§1.2.	Linear algebra over rings Algebraic number fields p-adic numbers	1 5 17
Chap	oter 2. Classical L-functions and Eisenstein series	25
§2.2. §2.3. §2.4. §2.5. §2.6.	Euler's method Analytic continuation and the functional equation Hurwitz and Dirichlet L-functions Shintani L-functions L-functions of real quadratic field L-functions of imaginary quadratic fields Hecke L-functions of number fields	25 33 40 47 54 63 66
Chap	oter 3. p-adic Hecke L-functions	73
§3.2. §3.3. §3.4. §3.5. §3.6. §3.7. §3.8.	Interpolation series Interpolation series in p-adic fields p-adic measures on \mathbb{Z}_p The p-adic measure of the Riemann zeta function p-adic Dirichlet L -functions Group schemes and formal group schemes Toroidal formal groups and p-adic measures p-adic Shintani L -functions of totally real fields p-adic Hecke L -functions of totally real fields	73 75 78 80 82 89 96 99
Chap	oter 4. Homological interpretation	107
§4.2. §4.3.	Cohomology groups on $G_m(C)$ Cohomological interpretation of Dirichlet L -values p-adic measures and locally constant functions Another construction of p-adic Dirichlet L -functions	107 117 118 120
Chap	oter 5. Elliptic modular forms and their <i>L</i> -functions	125
§5.2. §5.3. §5.4.	Classical Eisenstein series of GL(2) _{/Q} Rationality of modular forms Hecke operators The Petersson inner product and the Rankin product Standard <i>L</i> -functions of holomorphic modular forms	125 131 139 150 157
Chap	oter 6. Modular forms and cohomology groups	160
§6.2. §6.3. §6.4.	Cohomology of modular groups Eichler-Shimura isomorphisms Hecke operators on cohomology groups Algebraicity theorem for standard L-functions of GL(2) Mazur's p-adic Mellin transforms	160 167 175 186 189

vi Contents

Chapter 7. Ordinary A-adic forms, two variable p-adic Rankin products			
and Galois representations	194		
§7.1. p-Adic families of Eisenstein series	195		
§7.2. The projection to the ordinary part	200		
§7.3. Ordinary Λ-adic forms	208		
§7.4. Two variable p-adic Rankin product	221		
§7.5. Ordinary Galois representations into $GL_2(\mathbf{Z}_p[[X]])$	228		
§7.6. Examples of Λ-adic forms	234		
Chapter 8. Functional equations of Hecke L -functions	239		
§8.1. Adelic interpretation of algebraic number theory	239		
§8.2. Hecke characters as continuous idele characters	245		
§8.3. Self-duality of local fields	249		
§8.4. Haar measures and the Poisson summation formula	253		
§8.5. Adelic Haar measures §8.6. Functional equations of Hecke <i>L</i> -functions	257		
80.0. I unctional equations of Fiecke L-functions	261		
Chapter 9. Adelic Eisenstein series and Rankin products	272		
§9.1. Modular forms on GL ₂ (F _A)	272		
§9.2. Fourier expansion of Eisenstein series	282		
§9.3. Functional equation of Eisenstein series	292		
§9.4. Analytic continuation of Rankin products §9.5. Functional equations of Rankin products	298		
-	306		
Chapter 10. Three variable p-adic Rankin products	310		
§10.1. Differential operators of Maass and Shimura	310		
§10.2. The algebraicity theorem of Rankin products	317		
§10.3. Two variable Λ-adic Eisenstein series	326		
§10.4. Three variable p-adic Rankin products	331		
§10.5. Relation to two variable p-adic Rankin products	339		
§10.6. Concluding remarks	343		
Appendix: Summary of homology and cohomology theory	345		
References	365		
Answers to selected exercises			
Index	383		