

CONTENTS

1	Introduction	1
1.1	The Background of Machine Vision	1
1.2	The Aim of This Book	4
1.3	The Organization of This Book	6
1.3.1	Computational projective geometry	6
1.3.2	Translational motion, stereo and 3-D rotation	7
1.3.3	Analysis of 3-D motion and optical flow	9
1.3.4	Analysis of conics	11
1.3.5	Statistical analysis of geometric computation	12
1.4	Bibliographical Notes	13
2	Computational Projective Geometry, 1	15
2.1	Geometry of Perspective Projection	15
2.1.1	Interpretation of N-vectors	15
2.1.2	Vanishing points and vanishing lines	19
2.2	Computation of Points and Lines	20
2.2.1	Fundamental duality of N-vectors	20
2.2.2	Collinearity of points and concurrency of lines	21
2.3	Geometry of 2-D Projective Space	25
2.3.1	Collineations and camera rotation transformations	25
2.3.2	Correlations, polarities, conjugacy, and conics	28
2.4	Cross Ratio and Projective Coordinates	30
2.4.1	Perspective invariance of cross ratio	30
2.4.2	Projective invariance of cross ratio	35
2.4.3	Harmonic Range of Points	40
2.5	Bibliographical Notes	42
	Exercises	43
3	Computational Projective Geometry, 2	51
3.1	Geometry of Standard Polarity	51
3.1.1	The absolute conic and its polarity	51
3.1.2	Conjugacy and orthogonality	54
3.2	Camera Calibration	56

3.2.1	Determination of the focal length	56
3.2.2	Pose parameters and motion parameters	60
3.2.3	Constraints on rectangles and squares	63
3.3	3-D Road Shape Reconstruction	65
3.3.1	Modelling ideal roads	65
3.3.2	Local-flatness approximation	67
3.3.3	3-D reconstruction by curve fitting	69
3.4	Bibliographical Notes	72
	Exercises	74
4	Translational Motion and Stereo	77
4.1	Analysis of Translational Motion	77
4.1.1	N-velocities and trajectories	77
4.1.2	Focus of expansion	78
4.1.3	Constant velocity motion	80
4.1.4	Vanishing points and line orientations	82
4.2	Motion Parallax	84
4.2.1	Motion parallax of a point	84
4.2.2	Representation of a space line	86
4.2.3	Motion parallax of a line	89
4.2.4	Motion parallax for general motion	91
4.3	Analysis of Stereo	92
4.3.1	Epipolars and epipole	92
4.3.2	Disparity maps and depth maps	94
4.3.3	Converging stereo	95
4.4	Bibliographical Notes	96
	Exercises	98
5	Computation of 3-D Rotation	100
5.1	Representation of 3-D Rotation	100
5.1.1	Rotation matrices	100
5.1.2	Axis and angle of rotation	102
5.2	Optimal Estimation of 3-D Rotation	105
5.2.1	Least-squares estimation of 3-D rotation	105
5.2.2	Singular value decomposition	109
5.2.3	Polar decomposition	112
5.2.4	Quaternion representation	114
5.3	Orthogonality Recovery	117
5.3.1	Orthogonality fitting	117
5.3.2	Orthogonal frame reconstruction	118

5.3.3	Optimal resolution	121
5.4	Spherical Optimization Search	123
5.4.1	Optimization of pose and orientation	123
5.4.2	Quadratic search	124
5.4.3	Model update search	128
5.5	Bibliographical Notes	130
	Exercises	132
6	Analysis of 3-D Rigid Motion	143
6.1	Representation of Planar Surface Motion	143
6.1.1	Planar surface motion	143
6.1.2	Collineation of planar surface motion	146
6.2	3-D Interpretation of Planar Surface Motion	148
6.2.1	Analytical solution	148
6.2.2	Ambiguity of planar surface motion	151
6.3	Determination of Collineation	153
6.4	3-D Interpretation from Point Correspondence	156
6.4.1	General formulation	156
6.4.2	Optimization search	158
6.5	Least-Squares Point Correspondence Algorithm	160
6.5.1	Essential matrix and eight-point algorithm	160
6.5.2	Robust analytical solution	161
6.5.3	Decomposability and uniqueness	165
6.6	3-D Interpretation from Line Correspondence	166
6.6.1	General formulation	166
6.6.2	Optimization search	168
6.7	Least-Squares Line Correspondence Algorithm	170
6.7.1	Essential parameters and thirteen-line algorithm	170
6.7.2	Robust analytical solution	171
6.7.3	Uniqueness of the solution	177
6.8	Ambiguity of 3-D Interpretation	178
6.8.1	Critical surface for point correspondence	178
6.8.2	Degeneracy into two planar surfaces	183
6.8.3	Critical line congruence for line correspondence	185
6.9	Bibliographical Notes	188
	Exercises	191

7	Analysis of Optical Flow	196
7.1	Representation of Planar Surface Optical Flow	196
7.1.1	Infinitesimal surface motion	196
7.1.2	Optical flow and flow matrix	197
7.1.3	Optical flow of lines and dual flow	199
7.2	3-D Interpretation of Planar Surface Optical Flow	200
7.2.1	Optical flow and motion parameters	200
7.2.2	Analytical solution	201
7.3	Determination of the Flow Matrix	203
7.3.1	Flow-based approach	203
7.3.2	Contour-based approach	206
7.4	Representation of General Optical Flow	208
7.4.1	General optical flow equation	208
7.4.2	Twisted flow and the epipolar equation	210
7.5	3-D Interpretation of General Optical Flow	212
7.5.1	Least-squares algorithm	212
7.5.2	Optimization search	215
7.6	Critical Surface of Optical Flow	217
7.6.1	Critical surface equation	217
7.6.2	Degeneracy into two planes	220
7.7	Bibliographical Notes	221
	Exercises	223
8	Analysis of Conics	228
8.1	Conics and Their Canonical Forms	228
8.1.1	Representation of a conic	228
8.1.2	Canonical form of a conic	229
8.2	Polarity of a Conic	231
8.2.1	Poles, polars, and tangents	231
8.2.2	Conjugacy of points and lines	234
8.3	Intersections and Orthogonality	235
8.3.1	Intersections of a conic with a line	235
8.3.2	Interpretation of rectangular corners	237
8.4	Conic Fitting	240
8.4.1	Existence and uniqueness	240
8.4.2	Least-squares fitting	240
8.5	3-D Interpretation of a Conic	242
8.5.1	The supporting plane and the true shape	242
8.5.2	3-D interpretation of a circle	246

8.5.3	3-D interpretation of an ellipse	251
8.6	Mapping of Conics and Invisible Motions	258
8.6.1	Group of invisible motions	258
8.6.2	Mapping of conics	260
8.6.3	Standard circle	262
8.7	Invisible Optical Flows	263
8.7.1	Representation of invisible flows	263
8.7.2	Adjoint transformation of invisible flows	266
8.8	Deformation of a Conic	268
8.8.1	Linear space of conic deformations	268
8.8.2	Normal flow along a conic	271
8.9	3-D Interpretation of a Moving Conic	271
8.9.1	Finite motion of a conic	271
8.9.2	Infinitesimal motion of a conic	273
8.10	Bibliographical Notes	275
	Exercises	277
9	Statistical Analysis of Geometric Computa- tion, 1	280
9.1	Statistical Model of Noise	280
9.1.1	Covariance matrix of an N-vector	280
9.1.2	Model of noise	282
9.1.3	Effective focal length	283
9.2	Covariance Matrices of Joins and Intersections	284
9.3	Optimal Least-Squares Estimation	287
9.3.1	Optimal weights and optimal estimation	287
9.3.2	Covariance matrix of optimal estimation	289
9.3.3	Statistical bias of optimal estimation	292
9.4	Edge Fitting, Vanishing Points, and Focuses of Ex- pansion	294
9.4.1	Error in edge fitting	294
9.4.2	Error in vanishing points	297
9.4.3	Error in focuses of expansion	300
9.5	Statistics of Rotation Fitting	305
9.5.1	Covariance matrix of 3-D rotation	305
9.5.2	Covariance matrix of the best fitting rotation	305
9.6	Statistics of Depth from Stereo	309
9.6.1	Sources of error	309
9.6.2	Error due to image noise	310

9.6.3	Error due to uncertainty of camera orientation	311
9.6.4	Error due to uncertainty of base-line	312
9.7	Bibliographical Notes	314
	Exercises	315
10	Statistical Analysis of Geometric Computation, 2	318
10.1	Statistics of Focal Length Calibration	318
10.1.1	Reliability of focal length estimation	318
10.1.2	Optimal estimation of focal length	322
10.2	Statistical Analysis of 3-D Motion Estimation	326
10.2.1	Statistical bias of motion parameters	326
10.2.2	Small object approximation	330
10.2.3	Unbiased motion parameter estimation	333
10.3	Statistics of Conic Fitting	336
10.3.1	Optimal conic fitting	336
10.3.2	Covariance tensor of conic fitting	337
10.3.3	Statistical bias of conic fitting	340
10.3.4	Unbiased conic fitting	346
10.4	Hypothesizing and Testing Geometric Configurations	349
10.4.1	Gaussian approximation	349
10.4.2	Testing edge groupings	350
10.4.3	Testing vanishing points	352
10.4.4	Testing focuses of expansion	354
10.4.5	Testing vanishing lines	355
10.5	Bibliographical Notes	357
	Exercises	360
	References	364
	Answers	381
	Index	468