

Contents

Preface. A Personal View of Automated Reasoning Research xi

Chapter 1. Introduction to Automated Reasoning

1. Introduction	1
2. The potential scope of automated reasoning	1
3. The current scope of automated reasoning	3
4. Mathematical logic	4
5. Automation of first-order logic	4
6. Clauses	5
7. Conversion to clausal form	7
8. Cut rule	8
9. Substitutions	8
10. Unification algorithm	10
11. Binary resolution	11
12. Binary resolution is dead, long live binary resolution!	12
13. Hyperresolution	13
14. UR-resolution	13
15. Equality reasoning	14
16. Other strategies for fighting the combinatorial explosion	17
17. Infinite schemata	18
18. Theorem provers used in this work	19
19. Outline of remaining chapters	20
20. Survey of principal literature	22

Chapter 2. Von Neumann-Bernays-Gödel Set Theory

1. Introduction	23
2. Notation	25
3. Simplifications	26
3.1. Sethood	26
3.2. Equality	27
3.3. Introduction of ordered pairs	27
3.4. Use of ordered pairs	28
3.5. Constructor axioms	29
3.6. Class existence theorem	30
3.7. Constructors versus Skolem functors	32

4.	Clauses for axioms and definitions	34
5.	Proving classes equal	42
6.	Boolean demodulators	46
7.	Skolem functors and the Axiom of Choice	49
8.	Theorems proved	50
9.	Use of previously proved theorems	50
10.	Heuristics and option settings	51
11.	Proof finder or proof verifier?	53
12.	Proof of Cantor's theorem	54
13.	Proof that the composition of homomorphisms is a homomorphism	56
14.	Developing a unification algorithm appropriate to NBG set theory	63
15.	Conclusion	64

Chapter 3. Peano's Arithmetic

1.	Introduction	65
2.	The axioms	67
3.	Theorems proved	68
4.	Numerals	69
5.	Difference	70
6.	Ordering relation	70
7.	Induction	71
8.	Connection with NBG set theory	73
9.	Introduction of new functors	74
10.	Definition by primitive recursion	75
11.	Definition of lists	75
12.	A schema for proving metatheorems	77
13.	Greek classics 1: The square root of any prime is irrational	80
14.	Greek classics 2: There are infinitely many primes	82
15.	Unique factorization	84
16.	Prime power factorization of greatest common divisor	86
17.	Euler's theorem	88
18.	Related research	89
19.	Goldbach's conjecture	90

Chapter 4. Tarski's Geometry

1. Introduction	92
2. The axioms	93
3. Comparison with earlier system	96
4. Automated proof procedures	97
5. Canonicalization in absence of commutative unification	98
6. Theorems proved	99
7. Proofs of challenge theorems	106
8. Performance statistics	116
9. Related research	117
10. Further challenges	117
11. Discussion	118

Chapter 5. Löb's Theorem and Gödel's Two Incompleteness Theorems

1. Introduction	120
2. Background and motivation	120
3. Theorems of Gödel and Löb	122
4. The modal logic calculus $K4$	123
5. Formalization of $K4$ within OTTER	125
5.1. Tautologies	126
5.2. Comments on demodulators	129
6. Intermediate results	131
7. Löb's theorem	132
8. Gödel's first incompleteness theorem	134
9. Gödel's second incompleteness theorem	135
10. Further research	136
11. Conclusion	137

Chapter 6. Unsolved Problems in Elementary Number Theory

1. Introduction	138
2. The problems	139
3. Conclusion and beginning	150

Appendix 1. Gödel's Axioms for Set Theory	151
Appendix 2. Theorems Proved in NBG Set Theory	155
Appendix 3. Theorems Proved in Peano's Arithmetic	184
Bibliography	259
Index of Names	267
Index of Subjects	269