

# Contents

Preface	v
Acknowledgments	ix
CHAPTER I	
General Topology	1
1. Metric Spaces	1
2. Topological Spaces	3
3. Subspaces	8
4. Connectivity and Components	10
5. Separation Axioms	12
6. Nets (Moore–Smith Convergence) ✧	14
7. Compactness	18
8. Products	22
9. Metric Spaces Again	25
10. Existence of Real Valued Functions	29
11. Locally Compact Spaces	31
12. Paracompact Spaces	35
13. Quotient Spaces	39
14. Homotopy	44
15. Topological Groups	51
16. Convex Bodies	56
17. The Baire Category Theorem	57
CHAPTER II	
Differentiable Manifolds	63
1. The Implicit Function Theorem	63
2. Differentiable Manifolds	68
3. Local Coordinates	71
4. Induced Structures and Examples	72
	xi

5. Tangent Vectors and Differentials .....	76
6. Sard's Theorem and Regular Values .....	80
7. Local Properties of Immersions and Submersions .....	82
8. Vector Fields and Flows .....	86
9. Tangent Bundles .....	88
10. Embedding in Euclidean Space .....	89
11. Tubular Neighborhoods and Approximations .....	92
12. Classical Lie Groups ✧ .....	101
13. Fiber Bundles ✧ .....	106
14. Induced Bundles and Whitney Sums ✧ .....	111
15. Transversality ✧ .....	114
16. Thom–Pontryagin Theory ✧ .....	118

### CHAPTER III

<b>Fundamental Group</b> .....	<b>127</b>
1. Homotopy Groups .....	127
2. The Fundamental Group .....	132
3. Covering Spaces .....	138
4. The Lifting Theorem .....	143
5. The Action of $\pi_1$ on the Fiber .....	146
6. Deck Transformations .....	147
7. Properly Discontinuous Actions .....	150
8. Classification of Covering Spaces .....	154
9. The Seifert–Van Kampen Theorem ✧ .....	158
10. Remarks on $\text{SO}(3)$ ✧ .....	164

### CHAPTER IV

<b>Homology Theory</b> .....	<b>168</b>
1. Homology Groups .....	168
2. The Zeroth Homology Group .....	172
3. The First Homology Group .....	172
4. Functorial Properties .....	175
5. Homological Algebra .....	177
6. Axioms for Homology .....	182
7. Computation of Degrees .....	190
8. CW-Complexes .....	194
9. Conventions for CW-Complexes .....	198
10. Cellular Homology .....	200
11. Cellular Maps .....	207
12. Products of CW-Complexes ✧ .....	211
13. Euler's Formula .....	215
14. Homology of Real Projective Space .....	217
15. Singular Homology .....	219
16. The Cross Product .....	220
17. Subdivision .....	223
18. The Mayer–Vietoris Sequence .....	228
19. The Generalized Jordan Curve Theorem .....	230
20. The Borsuk–Ulam Theorem .....	240
21. Simplicial Complexes .....	245

22. Simplicial Maps .....	250
23. The Lefschetz–Hopf Fixed Point Theorem .....	253

CHAPTER V

Cohomology 260

1. Multilinear Algebra .....	260
2. Differential Forms .....	261
3. Integration of Forms .....	265
4. Stokes’ Theorem .....	267
5. Relationship to Singular Homology .....	269
6. More Homological Algebra .....	271
7. Universal Coefficient Theorems .....	281
8. Excision and Homotopy .....	285
9. de Rham’s Theorem .....	286
10. The de Rham Theory of $\mathbb{C}P^n$ ✧ .....	292
11. Hopf’s Theorem on Maps to Spheres ✧ .....	297
12. Differential Forms on Compact Lie Groups ✧ .....	304

CHAPTER VI

Products and Duality 315

1. The Cross Product and the Künneth Theorem .....	315
2. A Sign Convention .....	321
3. The Cohomology Cross Product .....	321
4. The Cup Product .....	326
5. The Cap Product .....	334
6. Classical Outlook on Duality ✧ .....	338
7. The Orientation Bundle .....	340
8. Duality Theorems .....	348
9. Duality on Compact Manifolds with Boundary .....	355
10. Applications of Duality .....	359
11. Intersection Theory ✧ .....	366
12. The Euler Class, Lefschetz Numbers, and Vector Fields ✧ .....	378
13. The Gysin Sequence ✧ .....	390
14. Lefschetz Coincidence Theory ✧ .....	393
15. Steenrod Operations ✧ .....	404
16. Construction of the Steenrod Squares ✧ .....	412
17. Stiefel–Whitney Classes ✧ .....	420
18. Plumbing ✧ .....	426

CHAPTER VII

Homotopy Theory 430

1. Cofibrations .....	430
2. The Compact-Open Topology .....	437
3. H-Spaces, H-Groups, and H-Cogroups .....	441
4. Homotopy Groups .....	443
5. The Homotopy Sequence of a Pair .....	445
6. Fiber Spaces .....	450
7. Free Homotopy .....	457
8. Classical Groups and Associated Manifolds .....	463

9. The Homotopy Addition Theorem .....	469
10. The Hurewicz Theorem .....	475
11. The Whitehead Theorem .....	480
12. Eilenberg–Mac Lane Spaces .....	488
13. Obstruction Theory $\star$ .....	497
14. Obstruction Cochains and Vector Bundles $\star$ .....	511
<b>Appendices</b>	
App. A. The Additivity Axiom .....	519
App. B. Background in Set Theory .....	522
App. C. Critical Values .....	531
App. D. Direct Limits .....	534
App. E. Euclidean Neighborhood Retracts .....	536
<b>Bibliography</b> .....	<b>541</b>
<b>Index of Symbols</b> .....	<b>545</b>
<b>Index</b> .....	<b>549</b>