

Contents

	Preface	ix
1	Exterior Algebra	1
	1.1 Exterior Powers of a Vector Space	1
	1.2 Multilinear Alternating Maps and Exterior Products	5
	1.3 Exercises	7
	1.4 Exterior Powers of a Linear Transformation	8
	1.5 Exercises	12
	1.6 Inner Products	13
	1.7 The Hodge Star Operator	17
	1.8 Exercises	20
	1.9 Some Formal Algebraic Constructions	21
	1.10 History and Bibliography	23
2	Exterior Calculus on Euclidean Space	24
	2.1 Tangent Spaces – the Euclidean Case	24
	2.2 Differential Forms on a Euclidean Space	28
	2.3 Operations on Differential Forms	31
	2.4 Exercises	33
	2.5 Exterior Derivative	35
	2.6 Exercises	39
	2.7 The Differential of a Map	41
	2.8 The Pullback of a Differential Form	43

2.9	Exercises	47
2.10	History and Bibliography	49
2.11	Appendix: Maxwell's Equations	50
3	Submanifolds of Euclidean Spaces	53
3.1	Immersions and Submersions	53
3.2	Definition and Examples of Submanifolds	55
3.3	Exercises	60
3.4	Parametrizations	61
3.5	Using the Implicit Function Theorem to Parametrize a Submanifold	64
3.6	Matrix Groups as Submanifolds	69
3.7	Groups of Complex Matrices	71
3.8	Exercises	72
3.9	Bibliography	75
4	Surface Theory Using Moving Frames	76
4.1	Moving Orthonormal Frames on Euclidean Space	76
4.2	The Structure Equations	78
4.3	Exercises	79
4.4	An Adapted Moving Orthonormal Frame on a Surface	81
4.5	The Area Form	85
4.6	Exercises	87
4.7	Curvature of a Surface	88
4.8	Explicit Calculation of Curvatures	91
4.9	Exercises	94
4.10	The Fundamental Forms: Exercises	95
4.11	History and Bibliography	97
5	Differential Manifolds	98
5.1	Definition of a Differential Manifold	98
5.2	Basic Topological Vocabulary	100
5.3	Differentiable Mappings between Manifolds	102
5.4	Exercises	104
5.5	Submanifolds	105
5.6	Embeddings	107

5.7	Constructing Submanifolds without Using Charts	110
5.8	Submanifolds-with-Boundary	111
5.9	Exercises	114
5.10	Appendix: Open Sets of a Submanifold	116
5.11	Appendix: Partitions of Unity	117
5.12	History and Bibliography	119
6	Vector Bundles	120
6.1	Local Vector Bundles	120
6.2	Constructions with Local Vector Bundles	122
6.3	General Vector Bundles	125
6.4	Constructing a Vector Bundle from Transition Functions	130
6.5	Exercises	132
6.6	The Tangent Bundle of a Manifold	134
6.7	Exercises	139
6.8	History and Bibliography	141
6.9	Appendix: Constructing Vector Bundles	141
7	Frame Fields, Forms, and Metrics	144
7.1	Frame Fields for Vector Bundles	144
7.2	Tangent Vectors as Equivalence Classes of Curves	147
7.3	Exterior Calculus on Manifolds	148
7.4	Exercises	151
7.5	Indefinite Riemannian Metrics	152
7.6	Examples of Riemannian Manifolds	153
7.7	Orthonormal Frame Fields	156
7.8	An Isomorphism between the Tangent and Cotangent Bundles	160
7.9	Exercises	161
7.10	History and Bibliography	163
8	Integration on Oriented Manifolds	164
8.1	Volume Forms and Orientation	164
8.2	Criterion for Orientability in Terms of an Atlas	167
8.3	Orientation of Boundaries	169
8.4	Exercises	172

8.5	Integration of an n -Form over a Single Chart	174
8.6	Global Integration of n -Forms	178
8.7	The Canonical Volume Form for a Metric	181
8.8	Stokes's Theorem	183
8.9	The Exterior Derivative Stands Revealed	184
8.10	Exercises	187
8.11	History and Bibliography	189
8.12	Appendix: Proof of Stokes's Theorem	189
9	Connections on Vector Bundles	194
9.1	Koszul Connections	194
9.2	Connections via Vector-Bundle-valued Forms	197
9.3	Curvature of a Connection	202
9.4	Exercises	206
9.5	Torsion-free Connections	212
9.6	Metric Connections	216
9.7	Exercises	219
9.8	History and Bibliography	222
10	Applications to Gauge Field Theory	223
10.1	The Role of Connections in Field Theory	223
10.2	Geometric Formulation of Gauge Field Theory	225
10.3	Special Unitary Groups and Quaternions	231
10.4	Quaternion Line Bundles	233
10.5	Exercises	238
10.6	The Yang–Mills Equations	242
10.7	Self-duality	244
10.8	Instantons	247
10.9	Exercises	249
10.10	History and Bibliography	250
	Bibliography	251
	Index	253