

Contents

Preamble	vii
1. The Basic Spaces	1
1.1 A Model for the Hyperbolic Plane	1
1.2 The Riemann Sphere $\overline{\mathbb{C}}$	7
1.3 The Boundary at Infinity of \mathbb{H}	16
2. The General Möbius Group	19
2.1 The Group of Möbius Transformations	19
2.2 Transitivity Properties of Möb^+	25
2.3 The Cross Ratio	30
2.4 Classification of Möbius Transformations	33
2.5 A Matrix Representation	36
2.6 Reflections	41
2.7 The Conformality of Elements of Möb	46
2.8 Preserving \mathbb{H}	49
2.9 Transitivity Properties of $\text{Möb}(\mathbb{H})$	54
3. Length and Distance in \mathbb{H}	57
3.1 Paths and Elements of Arc-length	57
3.2 The Element of Arc-length on \mathbb{H}	62
3.3 Path Metric Spaces	69
3.4 From Arc-length to Metric	73
3.5 Formulae for Hyperbolic Distance in \mathbb{H}	80
3.6 Isometries	83
3.7 Metric Properties of $(\mathbb{H}, d_{\mathbb{H}})$	89

4. Other Models of the Hyperbolic Plane	95
4.1 The Poincaré Disc Model	95
4.2 A General Construction	104
5. Convexity, Area, and Trigonometry	111
5.1 Convexity	111
5.2 A Characterization of Convex Sets	118
5.3 Hyperbolic Polygons	120
5.4 The Definition of Hyperbolic Area	130
5.5 Area and the Gauss-Bonnet Formula	133
5.6 Applications of the Gauss-Bonnet Formula	140
5.7 Trigonometry in the Hyperbolic Plane	146
6. Groups Acting on \mathbb{H}	153
6.1 The Geometry of the Action of $\text{Möb}(\mathbb{H})$	153
6.2 Discreteness	160
6.3 Fundamental Polygons	166
6.4 The Dirichlet Polygon	170
6.5 Poincaré's Theorem	174
Solutions	179
Further Reading	221
References	223
Notation	225
Index	228