Table of Contents

Preface xi

Introduction 1

Chapter 1 The Prime Number Theorem and Generalizations

§1 The Prime Number Theorem 5
§2 Primes in arithmetic progressions 15
§3 Dedekind's zeta function 19
§4 Hecke L-functions 21

Chapter 2 Artin L-functions

§1 Group-theoretic background 25
§2 Definition and basic properties of Artin L-functions 27
§3 The Aramata-Brauer theorem 30
§4 Dedekind's conjecture in the non-Galois case 32
§5 Zeros and poles of Artin L-functions 35
§6 Low order zeros of Dedekind zeta functions 37
§7 The Chebotarev Density Theorem 41
§8 Consequences of Artin's conjecture 46
§9 The least prime in a conjugacy class 52

Chapter 3 Equidistribution and L-functions

§1 Compact groups and Haar measures 65
§2 Weyl's criterion for equidistribution 66
§3 L-functions on G 67
§4 Deligne's prime number theorem 68

Chapter 4 Modular Forms and Dirichlet Series
§1 $SL_2(\mathbb{Z})$ and some of its subgroups 75
§2 The upper half plane 76
§3 Modular forms and cusp forms 77
§4 L-functions and Hecke's theorem 81
§5 Hecke operators 82
§6 Oldforms and Newforms 83
§7 The Sato-Tate conjecture 83
§8 Oscillations of Fourier coefficients of newforms 84
§9 Rankin's theorem 90

Chapter 5 Dirichlet L-functions
§1 Average value of $L(s,\chi)$ 93
§2 Polya-Vinogradov estimate 95
§3 Jutila's character sum estimate 97
§4 Average value of $L(\frac{1}{2},\chi_D)$ 104
§5 Non-vanishing for a positive proportion of characters, I 110
§6 Non-vanishing for a positive proportion of characters, II 119
§7 A conditional improvement 128

Chapter 6 Non-vanishing of Quadratic Twists of Modular L-functions
§1 Introduction 133
§2 The integrated Polya-Vinogradov estimate 141
§3 The main terms 142
§4 Estimates for real character sums 152
§5 Estimates for some weighted sums 158
§6 The statements $A^{\pm}(\alpha)$ and $C^{\pm}(\alpha)$ 160
§7 Proof of main result 170

Chapter 7 Selberg's Conjectures
§1 Selberg's class of Dirichlet series 177
§2 Basic consequences 180
§3 Artin's conjecture and Selberg's conjectures 181

Chapter 8 Suggestions for further reading 187

Author Index 192
Subject Index 194