

CONTENTS

0	Hyperbolic surfaces and Fuchsian groups: Summary	1
0.1	The hyperbolic plane and hyperbolic geometry	1
0.2	Hyperbolic surfaces	4
0.3	Aspects of finiteness of hyperbolic surfaces	5
0.4	Flexibility and geometrical boundedness	7
0.5	Action and deformation at infinity	9
0.6	The geometric limit	12
1	Hyperbolic 3-manifolds	15
1.1	The 3-dimensional hyperbolic space	15
1.1.1	The unit ball model and Möbius transformations	15
1.1.2	The upper half-space model and the projective model	19
1.2	Hyperbolic 3-manifolds and Kleinian models	23
1.2.1	Hyperbolic manifolds and developing maps	23
1.2.2	Kleinian groups	25
1.3	Examples of hyperbolic manifolds	30
1.3.1	Products and handlebodies	30
1.3.2	Pasting polyhedra	32
1.3.3	How to construct closed 3-manifolds	37
2	The basis of Kleinian group theory	41
2.1	Fundamental facts	41
2.1.1	Limit sets and non-elementary groups	41
2.1.2	Fundamental polyhedra	44
2.1.3	The action on the sphere at infinity	46
2.1.4	Fundamental sets on the sphere at infinity	49
2.2	The Margulis constant and cuspidal parts	52
2.2.1	The Jørgensen inequality	52
2.2.2	The thin parts of hyperbolic manifolds	56
2.3	The Selberg lemma	59
2.4	Construction of Kleinian groups	62
2.4.1	Combination and decomposition theorems	62
2.4.2	Examples of Kleinian groups	67
3	Geometrically finite Kleinian groups	73
3.1	Definitions of geometric finiteness	73
3.1.1	The definition using the convex core	73
3.1.2	The conical limit set	75
3.1.3	Equivalent conditions to geometric finiteness	78
3.2	Basic properties	80
3.2.1	Inheritance of geometric finiteness by subgroups	80

3.2.2	The Ahlfors measure zero theorem	84
3.3	Deformation and rigidity	87
3.3.1	Quasiconformal maps and quasi-isometries	87
3.3.2	Isomorphism theorems	91
3.3.3	The Mostow rigidity theorem	97
4	Finitely generated Kleinian groups	102
4.1	Finiteness theorems for Kleinian manifolds	102
4.1.1	The relative core theorem	102
4.1.2	Topological finiteness theorems	105
4.2	The Ahlfors finiteness theorem	108
4.2.1	Preparation from the theory of Teichmüller spaces	109
4.2.2	Proof of the Ahlfors finiteness theorem	112
4.3	Algebraic representation of qc deformation	114
4.3.1	$\mathrm{PSL}_2(\mathbb{C})$ -representation space	114
4.3.2	Schottky space	118
4.3.3	Quasifuchsian space and Bers slice	120
4.3.4	Projective structures	122
4.4	B -groups	124
4.4.1	Kleinian manifolds of B -groups	124
4.4.2	Boundary groups	127
5	The sphere at infinity	131
5.1	Conservative Kleinian groups	131
5.1.1	Action on the limit set	131
5.1.2	Dirichlet sets and horospherical limit sets	133
5.1.3	Sufficient conditions for conservativity	137
5.2	The Sullivan rigidity theorem	139
5.2.1	Divergent groups and ergodicity	139
5.2.2	Extension of the rigidity theorem	142
5.2.3	Proof of the Sullivan rigidity theorem	144
5.3	The Teichmüller space of the hyperbolic structures	152
5.3.1	Quasiconformal deformation and Teichmüller spaces	152
5.3.2	Quasi-isometric deformation space	155
5.4	Stability	160
5.4.1	Quasiconformal stability	160
5.4.2	Structural stability	164
6	Infinite ends of hyperbolic manifolds	168
6.1	Measured laminations and pleated surfaces	168
6.1.1	Measured laminations	168
6.1.2	The Thurston compactification	173
6.1.3	Realizable laminations in pleated surfaces	176
6.2	A step to the uniformization theorem	180
6.2.1	The double limit theorem	180
6.2.2	Hyperbolic structure of a mapping torus	183

6.3	Geometrically tame Kleinian groups	187
6.3.1	Tameness of infinite ends	187
6.3.2	The ending lamination	189
6.3.3	The Bonahon theorem	194
6.3.4	Thurston's theorem on the Ahlfors conjecture	199
7	Algebraic and geometric convergence	204
7.1	Geometric convergence	204
7.1.1	Definitions of geometric convergence	204
7.1.2	Geometric convergence of representations	208
7.2	Difference between algebraic and geometric limits	210
7.2.1	Dehn surgery	210
7.2.2	Iteration of modular transformations	215
7.2.3	The algebraic limit in the geometric limit	219
7.3	Strong convergence	221
7.3.1	Carathéodory convergence	221
7.3.2	The covering theorem	224
7.3.3	Conditions for strong convergence	226
7.3.4	A consequence of strong convergence	230
7.4	Algebraic convergence and qc deformation space	230
7.4.1	Projective structures and representations	230
7.4.2	Global geometry of quasiconformal deformation spaces	234
	Appendix	240
	References	244
	Index	251