

Contents

Preface	xi
Chapter 1. Metric Spaces	1
§1.1. Definitions	1
§1.2. Examples	3
§1.3. Metrics and Topology	7
§1.4. Lipschitz Maps	9
§1.5. Complete Spaces	10
§1.6. Compact Spaces	13
§1.7. Hausdorff Measure and Dimension	17
Chapter 2. Length Spaces	25
§2.1. Length Structures	25
§2.2. First Examples of Length Structures	30
§2.3. Length Structures Induced by Metrics	33
§2.4. Characterization of Intrinsic Metrics	38
§2.5. Shortest Paths	43
§2.6. Length and Hausdorff Measure	53
§2.7. Length and Lipschitz Speed	54
Chapter 3. Constructions	59
§3.1. Locality, Gluing and Maximal Metrics	59
§3.2. Polyhedral Spaces	67
§3.3. Isometries and Quotients	75

§3.4. Local Isometries and Coverings	78
§3.5. Arcwise Isometries	86
§3.6. Products and Cones	88
Chapter 4. Spaces of Bounded Curvature	101
§4.1. Definitions	101
§4.2. Examples	109
§4.3. Angles in Alexandrov Spaces and Equivalence of Definitions	114
§4.4. Analysis of Distance Functions	119
§4.5. The First Variation Formula	121
§4.6. Nonzero Curvature Bounds and Globalization	126
§4.7. Curvature of Cones	131
Chapter 5. Smooth Length Structures	135
§5.1. Riemannian Length Structures	136
§5.2. Exponential Map	150
§5.3. Hyperbolic Plane	154
§5.4. Sub-Riemannian Metric Structures	178
§5.5. Riemannian and Finsler Volumes	192
§5.6. Besikovitch Inequality	201
Chapter 6. Curvature of Riemannian Metrics	209
§6.1. Motivation: Coordinate Computations	211
§6.2. Covariant Derivative	214
§6.3. Geodesic and Gaussian Curvatures	221
§6.4. Geometric Meaning of Gaussian Curvature	226
§6.5. Comparison Theorems	237
Chapter 7. Space of Metric Spaces	241
§7.1. Examples	242
§7.2. Lipschitz Distance	249
§7.3. Gromov–Hausdorff Distance	251
§7.4. Gromov–Hausdorff Convergence	260
§7.5. Convergence of Length Spaces	265
Chapter 8. Large-scale Geometry	271
§8.1. Noncompact Gromov–Hausdorff Limits	271
§8.2. Tangent and Asymptotic Cones	275

§8.3. Quasi-isometries	277
§8.4. Gromov Hyperbolic Spaces	284
§8.5. Periodic Metrics	298
Chapter 9. Spaces of Curvature Bounded Above	307
§9.1. Definitions and Local Properties	308
§9.2. Hadamard Spaces	324
§9.3. Fundamental Group of a Nonpositively Curved Space	338
§9.4. Example: Semi-dispersing Billiards	341
Chapter 10. Spaces of Curvature Bounded Below	351
§10.1. One More Definition	352
§10.2. Constructions and Examples	354
§10.3. Toponogov's Theorem	360
§10.4. Curvature and Diameter	364
§10.5. Splitting Theorem	366
§10.6. Dimension and Volume	369
§10.7. Gromov-Hausdorff Limits	376
§10.8. Local Properties	378
§10.9. Spaces of Directions and Tangent Cones	390
§10.10. Further Information	398
Bibliography	405
Index	409