

Contents

Preface	xi
Projects	xiv
Chapter 1. Chain Complexes, Homology, and Cohomology	1
§1.1. Chain complexes associated to a space	1
§1.2. Tensor products, adjoint functors, and Hom	8
§1.3. Tensor and Hom functors on chain complexes	12
§1.4. Singular cohomology	14
§1.5. The Eilenberg-Steenrod axioms	19
§1.6. Projects for Chapter 1	22
Chapter 2. Homological Algebra	23
§2.1. Axioms for Tor and Ext; projective resolutions	23
§2.2. Projective and injective modules	29
§2.3. Resolutions	33
§2.4. Definition of Tor and Ext - existence	35
§2.5. The fundamental lemma of homological algebra	36
§2.6. Universal coefficient theorems	43
§2.7. Projects for Chapter 2	49
Chapter 3. Products	51

§3.1. Tensor products of chain complexes and the algebraic Künneth theorem	51
§3.2. The Eilenberg-Zilber maps	54
§3.3. Cross and cup products	56
§3.4. The Alexander-Whitney diagonal approximation	64
§3.5. Relative cup and cap products	67
§3.6. Projects for Chapter 3	70
Chapter 4. Fiber Bundles	77
§4.1. Group actions	77
§4.2. Fiber bundles	78
§4.3. Examples of fiber bundles	81
§4.4. Principal bundles and associated bundles	84
§4.5. Reducing the structure group	89
§4.6. Maps of bundles and pullbacks	90
§4.7. Projects for Chapter 4	92
Chapter 5. Homology with Local Coefficients	95
§5.1. Definition of homology with twisted coefficients	96
§5.2. Examples and basic properties	98
§5.3. Definition of homology with a local coefficient system	103
§5.4. Functoriality	105
§5.5. Projects for Chapter 5	108
Chapter 6. Fibrations, Cofibrations and Homotopy Groups	111
§6.1. Compactly generated spaces	111
§6.2. Fibrations	114
§6.3. The fiber of a fibration	116
§6.4. Path space fibrations	120
§6.5. Fiber homotopy	123
§6.6. Replacing a map by a fibration	123
§6.7. Cofibrations	127

§6.8.	Replacing a map by a cofibration	131
§6.9.	Sets of homotopy classes of maps	134
§6.10.	Adjoint of loops and suspension; smash products	136
§6.11.	Fibration and cofibration sequences	138
§6.12.	Puppe sequences	141
§6.13.	Homotopy groups	143
§6.14.	Examples of fibrations	145
§6.15.	Relative homotopy groups	152
§6.16.	The action of the fundamental group on homotopy sets	155
§6.17.	The Hurewicz and Whitehead theorems	160
§6.18.	Projects for Chapter 6	163
Chapter 7.	Obstruction Theory and Eilenberg-MacLane Spaces	165
§7.1.	Basic problems of obstruction theory	165
§7.2.	The obstruction cocycle	168
§7.3.	Construction of the obstruction cocycle	169
§7.4.	Proof of the extension theorem	172
§7.5.	Obstructions to finding a homotopy	175
§7.6.	Primary obstructions	176
§7.7.	Eilenberg-MacLane spaces	177
§7.8.	Aspherical spaces	183
§7.9.	CW-approximations and Whitehead's theorem	185
§7.10.	Obstruction theory in fibrations	189
§7.11.	Characteristic classes	191
§7.12.	Projects for Chapter 7	192
Chapter 8.	Bordism, Spectra, and Generalized Homology	195
§8.1.	Framed bordism and homotopy groups of spheres	196
§8.2.	Suspension and the Freudenthal theorem	202
§8.3.	Stable tangential framings	204
§8.4.	Spectra	210

§8.5. More general bordism theories	213
§8.6. Classifying spaces	217
§8.7. Construction of the Thom spectra	219
§8.8. Generalized homology theories	227
§8.9. Projects for Chapter 8	234
Chapter 9. Spectral Sequences	237
§9.1. Definition of a spectral sequence	237
§9.2. The Leray-Serre-Atiyah-Hirzebruch spectral sequence	241
§9.3. The edge homomorphisms and the transgression	245
§9.4. Applications of the homology spectral sequence	249
§9.5. The cohomology spectral sequence	254
§9.6. Homology of groups	261
§9.7. Homology of covering spaces	264
§9.8. Relative spectral sequences	266
§9.9. Projects for Chapter 9	266
Chapter 10. Further Applications of Spectral Sequences	267
§10.1. Serre classes of abelian groups	267
§10.2. Homotopy groups of spheres	276
§10.3. Suspension, looping, and the transgression	279
§10.4. Cohomology operations	283
§10.5. The mod 2 Steenrod algebra	288
§10.6. The Thom isomorphism theorem	295
§10.7. Intersection theory	299
§10.8. Stiefel-Whitney classes	306
§10.9. Localization	312
§10.10. Construction of bordism invariants	317
§10.11. Projects for Chapter 10	319
Chapter 11. Simple-Homotopy Theory	323
§11.1. Introduction	323

§11.2. Invertible matrices and $K_1(R)$	326
§11.3. Torsion for chain complexes	334
§11.4. Whitehead torsion for CW-complexes	343
§11.5. Reidemeister torsion	346
§11.6. Torsion and lens spaces	348
§11.7. The s-cobordism theorem	357
§11.8. Projects for Chapter 11	357
Bibliography	359
Index	363